Pearson Edexcel

# Mark Scheme (Results)

# January 2023

## Pearson Edexcel International GCSE In Mathematics A (4MA1) Paper 1HR

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)

- dep – dependent
- indep – independent
- awrt – answer which rounds to
- eeoo – each error or omission

- **No working**

  If no working is shown then correct answers normally score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**

  If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review. If there is a choice of methods shown, mark the method that leads to the answer on the answer line; where no answer is given on the answer line, award the lowest mark from the methods shown.

  If there is no answer on the answer line then check the working for an obvious answer.
- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.
- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

| **International GCSE Maths** | | | | | | |
|---|---|---|---|---|---|---|
| Apart from questions 6, 13, 14, 15c, 18, 24 (where the mark scheme states otherwise) the correct answer, unless clearly obtained from an incorrect method, should be taken to imply a correct method. | | | | | | |
| **1** | (a) | | $48 < S \leqslant 54$ | 1 | B1 | Allow 48 – 54 oe |
| | (b) | (33 × 4) + (39 × 14) + (45 × 18) + (51 × 19) + (57 × 5)<br>or 132 + 546 + 810 + 969 + 285 (= 2742)<br><br>[lower bound products are: 120, 504, 756, 912, 270]<br>[upper bound products are: 144, 588, 864, 1026, 300] | | 4 | M2 | M2 for at least **4** correct products added (need not be evaluated) **or**<br><br>If not M2 then award:<br><br>M1 for consistent use of value within interval (including end points) for at least **4** products which must be added<br><br>or<br><br>correct midpoints used for at least **4** products and not added |
| | | $\frac{"2742"}{60}$ | | | M1 | dep on M1<br>Allow division by their $\Sigma f$ provided addition or total under column seen |
| | | *Correct answer scores full marks (unless from obvious incorrect working)* | 45.7 | | A1oe | $45\frac{7}{10}$ or $\frac{457}{10}$<br>(accept 46 from correct working) |
| | | | | | | **Total 5 marks** |

| | | | | | |
|---|---|---|---|---|---|
| **2** | | | 3 | M1 | For area of 2 different faces (ie not 2 triangles) |
| | 0.5 × 4.8 × 3.6 (= 8.64) oe or 4.8 × 3.6 if clear intention for this to be 2 triangles<br>7 × 3.6 ( = 25.2)<br>7 × 4.8 (= 33.6)<br>7 × 6 (= 42)<br>(**all measurements with intention to add**) | | | M1 | For adding together 5 areas , at least 4 of which are correct<br><br>NB: (3.6 + 4.8 + 6) × 7 (= 100.8) is 3 faces |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | 118 | | A1 | 118.1 or 118.08 |
| | | | | | **Total 3 marks** |

| | | | | | |
|---|---|---|---|---|---|
| **3** | | ($x$ =) 3 | 3 | B1 | |
| | | ($y$ =) 6 | | B1 | |
| | | (z =) 10 | | B1 | |
| | | | | | **Total 3 marks** |

| | | | | | |
|---|---|---|---|---|---|
| **4** | 1600 × 0.16 (= 256) oe or 1 – 0.16 (= 0.84) oe | | 4 | M1 | |
| | 1600 – "256" or 1600 × "0.84" (= 1344) | | | M1 | |
| | $\frac{"1344"}{1400}(=0.96)$ **or** $\frac{1400-"1344"}{1400}(=0.04)$ **or** $\frac{"1344"}{1400}\times 100(=96)$ **or** $\frac{1400-"1344"}{1400}\times 100$ | | | M1 | |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | 4 | | A1 | SCB1 for 1856 seen if no other marks awarded |
| | | | | | **Total 4 marks** |

| | | | | | |
|---|---|---|---|---|---|
| **5** | $390 \div (8 - 2)$ (= 65) or<br>$\frac{8}{15} - \frac{2}{15} = 390$ or $\frac{8}{15}x - \frac{2}{15}x = 390$ or<br>$\frac{6}{15} = 390$ or $\frac{6}{15}x = 390$ oe | | 3 | M1 | M2 for $\frac{390 \times 15}{6}$ oe |
| | "65" × (2 + 5 + 8) oe or<br>$\frac{1}{15} = 65$ or $\frac{1}{15}x = 65$ or $\frac{1}{5} = 195$ or $\frac{1}{5}x = 195$ | | | M1 or for 975 seen with further work and a different answer | |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | 975 | | A1 SCB1 for 52, 130, 208 or 390, 975, 1560 (or 2925) or 97.5, 243.75, 390 (or 731.25) | |
| | | | | | **Total 3 marks** |

| | | | | | |
|---|---|---|---|---|---|
| **6** | eg $4x + 8y = 60$ or $3x + 6y = 45$<br>$-\ \underline{4x - 6y = 4}$ $+\ \underline{4x - 6y = 4}$<br>$(14y = 56)$ $(7x = 49)$<br><br>eg $4x - 6\left(\frac{15 - x}{2}\right) = 4$ or $4(15 - 2y) - 6y = 4$ oe | | 3 | M1 | Correct method to eliminate $x$ or $y$: coefficients of $x$ or $y$ the same **and** correct operator to eliminate selected variable (condone any one arithmetic error in multiplication) **or** correctly writing $x$ or $y$ in terms of the other variable and correctly substituting. |
| | eg $x + 2 \times 4 = 15$ or $7 + 2 \times y = 15$ | | | M1 | dep correct method to find second variable using their value from a correct method to find first variable or for repeating above method to find second variable. |
| | *Working required* | $x = 7$, $y = 4$ | | A1 | dep on M1 |
| | | | | | **Total 3 marks** |

| | | | | | | |
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| **7** | (a) | | 0.000 0932 | 1 | B1 | |
| | (b) | | $2.4 \times 10^5$ | 2 | B2 | If not B2, then B1 for 240 000 or $24 \times 10^4$ oe or $2.4 \times 10^a$ $a \neq 5$ |
| | (c) | | $1.8 \times 10^{121}$ | 2 | B2 | If not B2, then B1 for $18 \times 10^{120}$ or $1.8 \times 10^b$ $b \neq 121$ |
| | | | | | | **Total 5 marks** |

| | | | | | | |
|---|---|---|---|---|---|---|
| **8** | (a) | | $3c^2(6cd^2-7)$ | 2 | B2 | fully correct **or**<br>B1 for a correct partial factorisation with at least two terms outside the bracket ie $3c(6c^2d^2-7c)$ or $c^2\left(18cd^2-21\right)$<br>**or** the fully correct factor outside the bracket with two terms inside the bracket and at most one mistake $3c^2(.....)$ |
| | (b) (i) | eg $(y\pm6)(y\pm3)$ or<br>$y(y+3)-6(y+3)$ or<br>$y(y-6)+3(y-6)$ | | 2 | M1 | or $(y+a)(y+b)$ where $ab=-18$ **or** $a+b=-3$<br>or<br>factorisation which expands to give 2 out of 3 correct terms |
| | | [allow use of $x$ rather than $y$] | $(y-6)(y+3)$ | | A1 | |
| | (ii) | | $6, -3$ | 1 | B1 | ft must come from their factors in (b)(i) |
| | | | | | | **Total 5 marks** |

8(b) As we have always done, (ii) must ft from (i)

**If they do nothing in (i) and then factorise and give the solutions in (ii) can we give marks retrospectively** – yes, as long as nothing in (i) – this could gain M1A1B1 (correct factorisation and correct solutions) or M1A0B1 (factorisation worthy of the method mark, but not correct and ft solutions from incorrect factorisation) or M0A0B1 (incorrect factorisation that is worthy of no marks and then answers which ft from their incorrect factorisation)

**What do we do if they give the incorrect factorisation in (i) and then start again in (ii), showing the correct factors and give the correct answers from their factorisation in (ii) as answers?** Award M0A0 in (i) and then B1 in (ii)

**What do we do if nothing is done in (i) and then we see they have used the quadratic formula and got the answers from this in (ii)?** No marks at all M0A0B0

**What do we do if the student has got the correct factorisation in (i) and the correct answers in (ii) but also has the quadratic formula shown in (ii)?** We award M1A1B1 – assuming that the quadratic formula is a check

**What if they factorise and solve in part (i) with nothing in (ii)**

M1A1B1 if fully correct or M1A0B1(allowable factorisation) or M0A0B1 (ft from incorrect factorisation that is not allowable)

**What if they factorise in (i) and give the correct answers for (ii) in (i) and then a different answer for the solution in (ii)**

Award M1A1 in (i) and B0 in (ii)

**What if they factorise correctly and then expand and give the original expression on the answer line** – award full marks; the student knows how to factorise and is checking and gives their check as the answer.

| | | | | | |
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| **9** | $\frac{1}{2}\times 7\times h = 42$ oe or $(h=)\frac{42\times 2}{7}$ (= 12) oe or<br>$3.5^2 + h^2 = y^2$ or $h = \sqrt{y^2 - 3.5^2}$ oe | | 4 | M1 | A correct equation involving the height or a correct expression for height – could be in terms of $y$ |
| | $y^2 = \left(\frac{7}{2}\right)^2 + (\text{"12"})^2$ oe or $\frac{1}{2}\times 7\times \text{"}\sqrt{y^2 - 3.5^2}\text{"} = 42$ oe | | | M1 | (indep) use of *their* height (any found value that they have called 'height') |
| | $y = \sqrt{\left(\frac{7}{2}\right)^2 + (\text{"12"})^2}$ oe | | | M1 | **all values must come from a correct method** |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | 12.5 | | A1 | oe eg $\frac{25}{2}$ |
| | | | | | **Total 4 marks** |

| | | | | | |
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| **10** | $\sin 52 = \frac{12 \div 2}{r}$ oe **or** $\frac{r}{\sin 90} = \frac{6}{\sin 52}$ oe<br><br>**or** $\cos(90-52) = \frac{12 \div 2}{r}$ oe<br>**or** $(r^2 =)(12 \div 2)^2 + \left(\frac{12 \div 2}{\tan 52}\right)^2$ oe $\left[r^2 = 6^2 + 4.687...^2\right]$<br>**or** $\frac{r}{\sin 38} = \frac{12}{\sin 104}$ oe | | 4 | M1 | A correct trig statement for the radius<br>use of tan must also include a correct Pythagoras statement. |
| | $r = \frac{6}{\sin 52}$ (=7.614) oe<br>**or** $r = \frac{6}{\cos 38}$ oe<br>**or** $(r =)\sqrt{(12 \div 2)^2 + \left(\frac{12 \div 2}{\tan 52}\right)^2}$ $\left[r = \sqrt{6^2 + 4.687...^2}\right]$ oe<br><br>**or** $\frac{12 \sin 38}{\sin 104}$ oe | | | M1 | A correct method to find the radius of the circle<br><br>use of tan must also use Pythagoras to find an expression for $r$ |
| | (Area =) $\pi \times ("7.61...")^2$ | | | M1 | the radius must come from a completely correct method |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | 182 | | A1 | Accept 181 - 183 |
| | | | | | **Total 4 marks** |

| | | | | | | |
|---|---|---|---|---|---|---|
| **11** | (a) | | 7, 17, 32, 64, 80 | 1 | B1 | values seen in table |
| | (b) | | | 2 | M1ft | for at least 4 points plotted correctly at end of interval **or** for all points plotted consistently within each interval of the associated frequency table (eg at 5, 15, 25, 35, 45 or 0, 10, 20, 30, 40) at the correct height. ft their table dep on one error only in the table |
| | | (NB: a 'bar chart' type graph scores zero marks) | correct cf graph | | A1 | All points plotted correctly at end of interval (tolerance 1 small square – **there is an overlay**) and joined with a curve or line segments<br>accept curve that is not joined at (0, 0). |
| | (c) | *Accept a single value in the range OR ft their cf graph* | 33 | 1 | B1ft | Accept a single value in range 32 – 34 **or** ft their cf graph |
| | (d) | NB: readings are 21 - 23 and 37 - 39 (but for this M1 these do not have to be correct if correct working is shown – eg lines or marks indicating use of CF 20 (or 20.25)and CF 60 (or 60.75) with an indication on the Time axis at the correct points (or they can just show the correct readings)) | | 2 | M1ft | For correct use of LQ and UQ and subtraction, ft from a cum freq graph provided method is shown – eg a line horizontally to the graph from readings of CF 20 and CF 60 to meet the graph and then a vertical line to the Time axis(even if wrongly read scale) **or** clear marks on the graph and Time axis that correspond to the correct readings **or** correct values from the Time axis |
| | | *Accept a single value in the range OR ft their cf graph* | 16 | | A1ft | Accept a single value in range 15 to 17 or ft from their cumulative frequency graph provided method is shown eg subtraction of values that would be correct for their graph |
| | | | | | | **Total 6 marks** |

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| **12** | $2^{-4x} = 2^5$ or $-4x = 5$ or $-\frac{4}{5}x = 1$ oe | | 2 | M1 |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | $-\frac{5}{4}$ | | A1 oe allow eg $\frac{5}{-4}$ |
| | | | | **Total 2 marks** |

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| **13** | eg 10 000$x$ = 3818.18…_<br>$\underline{\quad 100x = \quad 38.18\ldots}$<br><br>**or** 1000$x$ = 381.818…_<br>$\underline{\quad 10x = \quad 3.818\ldots}$<br><br>**or** 100$x$ = 38.1818…_<br>$\underline{\quad x = \quad 0.3818\ldots}$<br><br>oe | | 2 | M1 | For **selecting** 2 correct recurring decimals that when subtracted give a whole number or terminating decimal (37.8 or 378 or 3780 etc)<br>eg 10 000$x$ = 3818.18... and 100$x$ = 38.1818....<br>**or** 1000$x$ = 381.818… and 10$x$ = 3.81818…<br>**or** 100$x$ = 38.1818… and $x$ = 0.381818…<br>with intention to subtract.<br>(if recurring dots not shown then showing at least **one** of the numbers to at least 5 sf)<br>**or** $0.38+0.00\dot{1}\dot{8}$ **and** eg $100x=0.1818...$, $10\,000x=18.1818...$ with intention to subtract. |
| | eg 10 000$x$ – 100$x$ = 3818.18... – 38.1818...= 3780<br>(9900$x$ = 3780) **and** $\frac{3780}{9900}=\frac{21}{55}$<br>**or**<br>eg 1000$x$ – 10$x$ = 381.818... – 3.81818...= 378<br>(990$x$ = 378) **and** $\frac{378}{990}=\frac{21}{55}$<br>**or**<br>eg 100$x$ – $x$ = 38.1818... – 0.381818...= 37.8<br>(99$x$ = 37.8) **and** $\frac{37.8}{99}=\frac{21}{55}$<br>**or**<br>eg 10 000$x$ – 100$x$ = 18.1818... – 0.181818...= 18<br>**and** $0.38+\frac{18}{9900}=\frac{38\times99+18}{9900}=\frac{3780}{9900}=\frac{21}{55}$ oe | shown | | A1 | for completion to $\frac{21}{55}$ dep on M1<br>*(NB: this is a "use algebra to show that…" question, so we need to see algebra as well as seeing all the stages of working to award full marks)* |
| | | | | | **Total 2 marks** |

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| **14** | $\frac{0.515}{6.25}$ | | 2 | M1 | For either bound correct (used or seen) |
| | *Working required* | 0.0824 | | A1 | dep on M1 Allow $\frac{103}{1250}$ |
| | | | | | **Total 2 marks** |

| | | | | | | |
|---|---|---|---|---|---|---|
| **15** | (a) | <table><tr><td>*x*</td><td>−2</td><td>−1</td><td>−0.5</td><td>0</td><td>1</td><td>1.5</td><td>2</td></tr><tr><td>*y*</td><td>0</td><td></td><td></td><td>2</td><td></td><td></td><td>4</td></tr></table> | | 2 | B2 | (B1 for 1 or 2 correct) |
| | (b) | | correct curve | 2 | B2 | For correct smooth curve. (**there is an overlay for the curve – check the line now for (c)**)<br>If not B2, then B1 for at least 5 points plotted correctly ft from table dep on B1 or B2 in (a) |
| | (c) | $2x^3 - 6x + 4 = -3x$ **or** $x^3 - 3x + 2 = -\frac{3}{2}x$<br>or $y = -\frac{3}{2}x$ seen (allow $-\frac{3}{2}x$) | | 3 | M1 | |
| | | $y = -\frac{3}{2}x$<br>allow a correct line that intercepts with the curve eg of points on line (0, 0), (–1, 1.5), (–1.5, 2.25), (–2, 3) | | | M1 | a correct line that intercepts with the curve<br><br>(a correct line drawn implies M2) |
| | | *Answer dependent on a correct line being drawn* | (*x*=) −1.6 | | A1ft | accept −1.6 or −1.7 or ft their curve/line intercept **dep on a correct line being drawn**<br><br>NB: if *y* value given as well then M2 only |
| | | | | | | **Total 7 marks** |

| | | | | | | |
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| **16** | (a) | | –0.5 | 1 | B1 | oe eg $-\frac{1}{2}, \frac{-1}{2}, \frac{1}{-2}$, –1/2 |
| | (b) | $(3x-5)y=2$ **or** $(3y-5)x=2$ **or** $3xy-5y=2$ **or** $3xy-5x=2$ oe or $3y-5=\frac{2}{x}$ **or** $3x-5=\frac{2}{y}$ oe | | 2 | M1 | remove denominator or get to the stage $3y-5=\frac{2}{x}$ or $3x-5=\frac{2}{y}$ |
| | | *Correct answer scores full marks (unless from obvious incorrect working)* | $\frac{2+5x}{3x}$ | | A1oe | eg $\frac{2}{3x}+\frac{5}{3}$ or $\frac{\frac{2}{x}+5}{3}$ **must be in terms of *x*** |
| | (c) | $5\left(x^2-4x\right)$.....**or** $5\left(x^2-4x.....\right)$ **or** $5(x-2)^2\ldots$ | | 3 | M1 | |
| | | $5\left[(x-2)^2-(-2)^2\right]$.....**or** $5\left[(x-2)^2-(-2)^2.....\right]$ **or** $5(x-2)^2-20$ ……. **or** $5\left[(x-2)^2+\frac{3}{5}\right]$ | | | M1 | $(-2)^2$ can be $2^2$ or 4 or $\left(\pm\frac{4}{2}\right)^2$ |
| | | *Correct answer scores full marks (unless from obvious incorrect working)* | $5(x-2)^2+3$ | | A1 | |
| | | | | | | **Total 6 marks** |
| **Alternative mark scheme for 16c** | | | | | | |
| | | $ax^2-2abx+ab^2+c$ | | 3 | M1 | for multiplying out $a(x-b)^2+c$ to obtain $ax^2-2abx+ab^2+c$ oe |
| | | *2* of: $a=5$ $2ab=20$ oe $ab^2+c=23$ oe | | | M1 | for equating coefficients and making 2 correct statements |
| | | | $5(x-2)^2+3$ | | A1 | |

| | | | | | |
|---|---|---|---|---|---|
| **17** | $12 = \frac{1}{2} \times 4.6 \times 8.3 \times \sin ABC$ or $\frac{4.6h}{2} = 12$ $(h = 5.217....)$ | | 5 | M1 | a correct equation for the area to find angle *ABC* or to find the perpendicular height of the triangle. |
| | $ABC = \sin^{-1}\left(\frac{12}{\frac{1}{2} \times 4.6 \times 8.3}\right)$ (= 38.947…) oe or<br>$ABC = \sin^{-1}(0.6286)$ (= 38.947...) or<br>$ABC = \sin^{-1}\left(\frac{"5.217..."}{8.3}\right)$ (= 38.947...) or<br>$BM^2 = 8.3^2 - "5.217..."^2$ | | | M1 | A correct method to find angle *ABC*<br>or<br>a correct method to find $BM^2$ where *CMB* is 90º |
| | $AC^2 = 4.6^2 + 8.3^2 - 2 \times 4.6 \times 8.3 \times \cos("38.947")$ [allow cos39º]<br>or $AC^2 = 30.6(627...)$<br>$BM = \sqrt{8.3^2 - "5.217..."^2}$ (=6.455…) | | | M1 | a correct start to the cosine rule to find length AC<br>or a fully correct method for *BM* |
| | or $AC = \sqrt{"30.6(6...)"}$<br>**or**<br>5.5(3739…) | | | A1 | A correct value for *AC* which can be the square root of 30.6(6…) |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | 18.4 | | A1 | Allow answers in range 18.4 to 18.45 |
| | | | | | **Total 5 marks** |

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| **18** | $\sqrt{3}x - x = 6 + 2\sqrt{3}$ oe or $x - x\sqrt{3} = -6 - 2\sqrt{3}$<br><br>(allow $-2\sqrt{9}$ or $-2(\sqrt{3})^2$ for $-6$ or $2\sqrt{9}$ or $2(\sqrt{3})^2$ for $6$) | | 4 | M1 | expanding bracket and collecting terms. Condone one error |
| | $(x =)\ \dfrac{6+2\sqrt{3}}{\sqrt{3}-1}$ oe eg $\dfrac{-6-2\sqrt{3}}{1-\sqrt{3}}$ | | | A1 | oe must be a correct fraction with irrational numerator and denominator |
| | $(x =)\ \dfrac{(6+2\sqrt{3})}{(\sqrt{3}-1)} \times \dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$ or $\dfrac{(6+2\sqrt{3})(\sqrt{3}+1)}{2}$ oe or<br>$\dfrac{\left(6+2\sqrt{3}\right)}{\left(-1+\sqrt{3}\right)} \times \dfrac{\left(-1-\sqrt{3}\right)}{\left(-1-\sqrt{3}\right)}$ oe or<br>$\dfrac{\left(-6-2\sqrt{3}\right)\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)}$ oe | | | M1 | (indep) Multiplying the numerator and denominator of their fraction by $\sqrt{3}+1$ oe or showing 2 or –2 as the denominator and multiplying the numerator by $\sqrt{3}+1$ oe<br><br>or rationalising **their** denominator, so long as it is of the form $p+q\sqrt{3}$ where $p$ and $q$ are non zero integers<br><br>(condone missing brackets provided meaning is clear) |
| | *Working required* | $6+4\sqrt{3}$ | | A1 | dep on M1A1M1 with no errors seen |
| | | | | | **Total 4 marks** |

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| **19** (a) | $P=\frac{k}{y^2}$ | | 3 | M1 | oe (the constant term, *k*, can be any other letter apart from *a* or *P* or *y*) |
| | eg $a=\frac{k}{4^2}$ **or** $k=16a$ | | | M1 | oe |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | $P=\frac{16a}{y^2}$ | | A1 | oe eg $P=16ay^{-2}$ or $P=\frac{4^2a}{y^2}$ |
| (b) | $\sqrt{"\frac{16a}{4a}"}=c\sqrt{a}$ oe eg $\frac{16a}{4a}=c^2a$ or $4a=\frac{16a}{c^2a}$ **or** $4a\times c^2a=16a$ **oe** **or**<br>(when $P=4a$) $y^2=\frac{16a}{4a}$ **or** $y^2=4$ **or** $y=\sqrt{\frac{16a}{4a}}(=2)$ oe | | 3 | M1 | ft a correct formula involving the constant term (*c* used here) and *a* **or**<br>ft for an expression or value of $y^2$ or *y* given for when $P=4a$ |
| | $c=\sqrt{\frac{4}{a}}$ **or** $c=\frac{\pm2}{\sqrt{a}}$ **or** $c=\frac{\pm2\sqrt{a}}{a}$ oe<br>allow the constant term squared eg $c^2=\frac{16a}{4a^2}\left(=\frac{4}{a}\right)$ | | | M1 | (implies previous M1) a correct value, in terms of *a*, for the constant term or the constant term squared – need not be simplified |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | $P=\frac{4a^2}{x}$ | | A1 | oe eg $P=\frac{16a}{\frac{4x}{a}}$ or $P=\frac{16a^2}{4x}$ |
| | | | | | **Total 6 marks** |

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| **20** | | ($a$ =) 2<br>($b$ =) 135 | 2 | B2 | If not B2, then B1 for one correct value |
| | | | | | **Total 2 marks** |

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| **21** | $(AD=)\dfrac{2.2}{\tan 18}$ (= 6.77...) or<br>$(EA=)\dfrac{2.2}{\sin 18}(=7.11...)$ | | 4 | M1 | a correct method to find *AD* or *AE* |
| | $(DB=)\sqrt{("6.77...")^2+6^2}$ (= 9.04...) or<br>$(EB=)\sqrt{6^2+"7.11..."^2}$ (= 9.31...) or<br>$(EB=)\sqrt{6^2+"6.77..."^2+2.2^2}(=9.31...)$ | | | M1 | a correct method to find *DB* or *EB* |
| | $\tan DBE=\dfrac{2.2}{"9.04..."}$ or<br>$\sin DBE=\dfrac{2.2}{"9.31..."}$ or $\sin DBE=\dfrac{2.2\sin 90}{"9.31..."}$<br>$\cos DBE=\dfrac{"9.04..."}{"9.31..."}$ or use of cosine rule | | | M1 | complete method to find one of tan*DBE* or sin*DBE* or cos*DBE*–<br>NB: if using cosine, the student will need to have found *DB* and *EB* previously |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | 13.7 | | A1 | Allow answers in range 13.59 – 13.8 |
| | | | | | **Total 4 marks** |

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| **22** (a) | $\overrightarrow{ON} = \mathbf{b} + \frac{2}{5}(\mathbf{a} - \mathbf{b})$ oe or $\overrightarrow{ON} = \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$ oe | | 2 | M1 | |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | $\frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ | | A1 | oe eg $\frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$ but must be one term in **a** and one in **b** |
| (b) | $\overrightarrow{ME} = \frac{8}{5}\mathbf{a} - \frac{4}{5}\mathbf{b}$<br>$\overrightarrow{NE} = \frac{6}{5}\mathbf{a} - \frac{3}{5}\mathbf{b}$ *(all oe but simplified)*<br>$\overrightarrow{MN} = \frac{2}{5}\mathbf{a} - \frac{1}{5}\mathbf{b}$ | | 3 | M1ft | for one of $\overrightarrow{ME}$, $\overrightarrow{NE}$ or $\overrightarrow{MN}$ or one of $\overrightarrow{EM}$, $\overrightarrow{EN}$ or $\overrightarrow{NM}$<br>ft (dep on M1 in (a)) their expression for $\overrightarrow{ON}$<br>for this mark only [ $\overrightarrow{ME} = \overrightarrow{ON} + \frac{6}{5}\mathbf{a} - \frac{7}{5}\mathbf{b}$<br>$\overrightarrow{MN} = \overrightarrow{ON} - \frac{4}{5}\mathbf{b}, \overrightarrow{NE} = -\overrightarrow{ON} + \frac{11}{3}\mathbf{a}$ ] |
| | $\overrightarrow{ME} = \frac{8}{5}\mathbf{a} - \frac{4}{5}\mathbf{b}$<br>$\overrightarrow{NE} = \frac{6}{5}\mathbf{a} - \frac{3}{5}\mathbf{b}$ *(all oe but simplified)*<br>$\overrightarrow{MN} = \frac{2}{5}\mathbf{a} - \frac{1}{5}\mathbf{b}$ | | | M1 | for two of $\overrightarrow{ME}$, $\overrightarrow{NE}$ or $\overrightarrow{MN}$ or two of $\overrightarrow{EM}$, $\overrightarrow{EN}$ or $\overrightarrow{NM}$<br>**must be correct** |
| | *Evidence of a vector method needed* | shown | | A1 | eg $\overrightarrow{ME} = 4 \times \overrightarrow{MN}$ **or** $\overrightarrow{NE} = 3 \times \overrightarrow{MN}$ **or** $\overrightarrow{ME} = \frac{4}{3} \times \overrightarrow{NE}$<br>**or** showing they are multiples of the same vector eg<br>$\overrightarrow{MN} = \frac{1}{5}(2\mathbf{a} - \mathbf{b})$ and $\overrightarrow{NE} = \frac{3}{5}(2\mathbf{a} - \mathbf{b})$ |
| | | | | | **Total 5 marks** |

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| **23** | $\left(\frac{dy}{dx} =\right) 16x - 14$ | | 5 | M1 | Differentiation to obtain 2 terms with at least 1 correct |
| | $16x - 14 = 10$ | | | M1 | their d$y$/d$x$ = 10 dep on M1 |
| | (1.5, –9) or $x = 1.5$, $y = -9$ | | | A1 | coordinates of point on curve at which gradient is 10 – allow given as coordinates or as $x$ worked out and $y$ worked out if meaning is clear |
| | eg $y - -9 = -\frac{1}{10}(x - \frac{3}{2})$ oe or<br>eg $-9 = -\frac{1}{10} \times 1.5 + c$ oe | | | M1 | A correct method to find the equation for line **Q** using (1.5, –9) |
| | *Correct answer scores full marks (unless from obvious incorrect working)* | $2x + 20y + 177 = 0$ | | A1 | oe where $a$, $b$, $c$ are integers eg $10x + 100y + 885 = 0$ |
| | | | | | **Total 5 marks** |

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| **24** | $(S_{20}=)10[2A+19\times11]=10170$ oe (where $A$ is the $u_{(k-19)}$ th term) | | 5 | M1 | |
| | $A=\left(\frac{10170}{10}-19\times11\right)\div2\,(=404)$ | | | M1 | |
| | $8+(P-1)11=\text{"404"}$ oe (where $P$ is the number of terms from 20 to the end) | | | M1 | M2 for $8+11\times(k-20)=$"404" |
| | $P=\frac{\text{"404"}-8+11}{11}\,(=37)$ | | | M1 | |
| | *Working required* | 56 | | A1 | dep on M1 |
| | **ALTERNATIVE METHOD** | | | | |
| | $(S_k=)\frac{k}{2}[2\times8+(k-1)11]$ **or** $(S_{k-20}=)\frac{(k-20)}{2}[2\times8+(k-21)11]$ **or** $(u_{k-19}=)8+11(k-20)$ **or** $(u_k=)8+11(k-1)$ *(allow use of letter other than k)* | | 5 | M1 | for $S_k$ **or** $S_{k-20}$ **or** $u_k$ or $u_{k-19}$ **a and d must be substituted correctly** |
| | $(S_k=)\frac{k}{2}[2\times8+(k-1)11]$ **and** $(S_{k-20}=)\frac{(k-20)}{2}[2\times8+(k-21)11]$ **or** $(u_{k-19}=)8+11(k-20)$ **and** $(u_k=)8+11(k-1)$ | | | M1 | For correct expressions for both $S_k$ and $S_{k-20}$ **or** $u_k$ and $u_{k-19}$ |
| | $10170=\frac{k}{2}[\text{"16"}+(k-1)11]-\frac{(k-20)}{2}[\text{"16"}+(k-21)11]$ oe **or** $10170=\frac{20}{2}([8+11(k-20)+[8+11(k-1)])$ oe | | | M1 | |
| | eg $10170=160+\frac{11}{2}[40k-420]$ oe eg $440k = 24640$ or $2240 = 40k$ oe | | | M1 | Expanding to obtain a linear equation **and** collecting terms in $k$ |
| | *Working required* | 56 | | A1 | dep on M1 |
| | | | | | **Total 5 marks** |