



Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE
NAME

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CENTRE
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FURTHER MATHEMATICS

9231/11

Paper 1

May/June 2017

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **27** printed pages and **1** blank page.

BLANK PAGE

3

1 It is given that $\sum_{r=1}^n u_r = n^2(2n + 3)$, where n is a positive integer.

(i) Find $\sum_{r=n+1}^{2n} u_r$. [2]

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(ii) Find u_r . [3]

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[illegible]

5**3** A curve C has equation $\tan y = x$, for $x > 0$.**(i)** Use implicit differentiation to show that

$$\frac{d^2y}{dx^2} = -2x \left(\frac{dy}{dx} \right)^2. \quad [3]$$

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(ii) Hence find the value of $\frac{d^2y}{dx^2}$ at the point $(1, \frac{1}{4}\pi)$ on C . [2]

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4 (i) Find the value of k for which the set of linear equations

$$\begin{aligned}x + 3y + kz &= 4, \\4x - 2y - 10z &= -5, \\x + y + 2z &= 1,\end{aligned}$$

[3]

[illegible]

where $\mathbf{a}$ and $\mathbf{b}$ are vectors and t is a scalar. [3]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

5 The matrix $\mathbf{A}$, given by

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & -2 \\ 6 & 4 & -6 \\ 6 & 5 & -7 \end{pmatrix},$$

has eigenvalues 1, -1 and -2 .

(i) Find a set of corresponding eigenvectors.

[4]

[illegible]

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(ii) The matrix **B** is given by $\mathbf{B} = \mathbf{A} - 2\mathbf{I}$, where **I** is the 3×3 identity matrix. Write down the eigenvalues of **B**, and state a set of corresponding eigenvectors. [2]

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6 Let $I_n = \int_0^{\frac{1}{2}\pi} x^n \sin x \, dx$.

(i) Prove that, for $n \geq 2$,

$$I_n + n(n-1)I_{n-2} = n\left(\frac{1}{2}\pi\right)^{n-1}. \quad [4]$$

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

7 By finding a cubic equation whose roots are α , β and γ , solve the set of simultaneous equations

$$\begin{aligned}\alpha + \beta + \gamma &= -1, \\ \alpha^2 + \beta^2 + \gamma^2 &= 29, \\ \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} &= -1.\end{aligned}\tag{8}$$

[illegible]

8 (i) Let $z = \cos \theta + i \sin \theta$. Show that $z - \frac{1}{z} = 2i \sin \theta$ and hence express $16 \sin^5 \theta$ in the form $\sin 5\theta + p \sin 3\theta + q \sin \theta$, where p and q are integers to be determined. [6]

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This image shows a full page of a document template. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings present.

9 The curve C has equation $y = \frac{x^2 - 3x + 6}{1 - x}$.

(i) Find the equations of the asymptotes of C . [3]

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(ii) Find the coordinates of the turning points of C . [3]

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(iii) Find the coordinates of any intersections with the coordinate axes. [2]

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(iv) Sketch C . [3]

10 It is given that $x = t^{\frac{1}{2}}$, where $x > 0$ and $t > 0$, and y is a function of x .

(i) Show that $\frac{dy}{dx} = 2t^{\frac{1}{2}} \frac{dy}{dt}$ and $\frac{d^2y}{dx^2} = 2 \frac{dy}{dt} + 4t \frac{d^2y}{dt^2}$. [3]

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(ii) Hence show that the differential equation

$$\frac{d^2y}{dx^2} - \left(8x + \frac{1}{x}\right) \frac{dy}{dx} + 12x^2y = 4x^2e^{-x^2} \qquad (*)$$

reduces to the differential equation

$$\frac{d^2y}{dt^2} - 4 \frac{dy}{dt} + 3y = e^{-t}. \qquad [1]$$

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(iii) Find the general solution of (*), giving y in terms of x . [7]

This image shows a full page of a document template. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings present.

11 The curve C has polar equation $r = a(1 + \sin \theta)$ for $-\pi < \theta \leq \pi$, where a is a positive constant.

(i) Sketch C . [2]

(ii) Find the area of the region enclosed by C . [4]

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$$s = (\sqrt{2})a \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \sqrt{1 + \sin \theta} \, d\theta. \quad [3]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]

12 Answer only **one** of the following two alternatives.

EITHER

The curve C has equation $y = \frac{1}{2}(e^x + e^{-x})$ for $0 \leq x \leq 4$.

- (i) The region R is bounded by C , the x -axis, the y -axis and the line $x = 4$. Find, in terms of e , the coordinates of the centroid of the region R . [10]

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[illegible]

OR

The position vectors of the points A, B, C, D are

$$\mathbf{i} + \mathbf{j} + 3\mathbf{k}, \quad 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}, \quad 3\mathbf{i} - \mathbf{j} + \mathbf{k}, \quad 5\mathbf{i} - 5\mathbf{j} + \alpha\mathbf{k},$$

respectively, where α is a positive integer. It is given that the shortest distance between the line AB and the line CD is equal to $2\sqrt{2}$.

- (i) Show that the possible values of α are 3 and 5. [7]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (ii) Using $\alpha = 3$, find the shortest distance of the point D from the line AC , giving your answer correct to 3 significant figures. [3]

- (iii) Using $\alpha = 3$, find the acute angle between the planes ABC and ABD , giving your answer in degrees. [4]

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