

Cambridge
International
A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE
NAME

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CENTRE
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FURTHER MATHEMATICS

9231/12

Paper 1

October/November 2017

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **23** printed pages and **1** blank page.



This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3

2 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 4 - 5t^2. \quad [6]$$

[illegible]

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$$\frac{d^n}{dx^n}(x^n \ln x) = n! \left(\ln x + 1 + \frac{1}{2} + \dots + \frac{1}{n} \right). \quad [5]$$

This image shows a full page of white paper designed for handwriting practice. It features 18 evenly spaced, horizontal dashed lines that run across the entire width of the page. The lines are thin and black, providing a guide for letter height and placement. There are no margins, text, or other markings on the page.

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- 4** The cubic equation $2x^3 - 3x^2 + 4x - 10 = 0$ has roots α , β and γ .

(i) Find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. [4]

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(ii) Find the value of $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$. [4]

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5 The curve C has equation $2x^3 + 3x^2y - 3y^3 - 16 = 0$.

- [illegible]

[3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

(i) Find the area of the triangle ABC . [4]

[illegible]

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- (ii)** Find the perpendicular distance of the point A from the line BC . [3]

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- (iii)** Find the cartesian equation of the plane through A , B and C . [2]

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7 The linear transformation $T: \mathbb{R}^4 \rightarrow \mathbb{R}^4$ is represented by the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 & 3 \\ 5 & -3 & -4 & 25 \\ 6 & -4 & -6 & 28 \\ 7 & -5 & -8 & 31 \end{pmatrix}.$$

(i) Find the rank of $\mathbf{A}$ and a basis for the null space of $\mathbf{T}$. [7]

This image shows a full page of a document template designed for handwriting practice. It consists of approximately 20 evenly spaced, horizontal dashed lines extending across the entire width of the page. The background is plain white, and there are no margins, text, or other markings present.

(ii) Find the matrix product $\mathbf{A} \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$ and hence find the general solution of the equation $\mathbf{A}\mathbf{x} = \begin{pmatrix} 3 \\ 21 \\ 24 \\ 27 \end{pmatrix}$.

8 Let $I_n = \int_0^{\frac{1}{4}\pi} \sec^n x \, dx$ for $n > 0$.

(i) Find the value of I_2 . [2]

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(ii) Show that, for $n > 2$,

$$(n-1)I_n = 2^{\frac{1}{2}n-1} + (n-2)I_{n-2}. \quad [5]$$

[illegible]

- (iii) The curve C has equation $y = \sec^3 x$ for $0 \leq x \leq \frac{1}{4}\pi$. The region R is bounded by C , the x -axis, the y -axis and the line $x = \frac{1}{4}\pi$. Find the volume of revolution generated when R is rotated through 2π radians about the x -axis. [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

$$y = \frac{3x - 9}{(x - 2)(x + 1)}.$$

- (i) Find the equations of the asymptotes of C . [2]

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- (ii)** Show that there is no point on C for which $\frac{1}{3} < y < 3$. [4]

This image shows a full page of a document template designed for handwritten notes or essays. It features a series of evenly spaced, horizontal grey lines that run across the entire width of the page. The lines are thin and light in color, providing a guide for writing without being distracting. There is no margin, header, or footer present on the page.

15**(iii)** Find the coordinates of the turning points of C .**[3]**

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(iv) Sketch C .**[3]**

$$\sin 5\theta = 5 \sin \theta - 20 \sin^3 \theta + 16 \sin^5 \theta. \quad [5]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

17

- (ii) Hence explain why the roots of the equation $16x^4 - 20x^2 + 5 = 0$ are $x = \pm \sin \frac{1}{5}\pi$ and $x = \pm \sin \frac{2}{5}\pi$.
[3]

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- (iii) Without using a calculator, find the exact values of

$$\sin \frac{1}{5}\pi \sin \frac{2}{5}\pi \sin \frac{3}{5}\pi \sin \frac{4}{5}\pi \quad \text{and} \quad \sin^2\left(\frac{1}{5}\pi\right) + \sin^2\left(\frac{2}{5}\pi\right).$$
 [4]

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11 Answer only **one** of the following two alternatives.

EITHER

- (i) The vector **e** is an eigenvector of the matrix **A**, with corresponding eigenvalue λ , and is also an eigenvector of the matrix **B**, with corresponding eigenvalue μ . Show that **e** is an eigenvector of the matrix **AB** with corresponding eigenvalue $\lambda\mu$. [3]

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- (ii) Find the eigenvalues and corresponding eigenvectors of the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 3 \\ 3 & 2 & -3 \\ 1 & 1 & 2 \end{pmatrix}.$$

[6]

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(iii) The matrix **B**, where

$$\mathbf{B} = \begin{pmatrix} 3 & 6 & 1 \\ 1 & -2 & -1 \\ 6 & 6 & -2 \end{pmatrix},$$

has eigenvectors $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Find the eigenvalues of the matrix **AB**, and state corresponding eigenvectors. [4]

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21**OR**

The polar equation of a curve C is $r = a(1 + \cos \theta)$ for $0 \leq \theta < 2\pi$, where a is a positive constant.

(i) Sketch C .

[2]

(ii) Show that the cartesian equation of C is

$$x^2 + y^2 = a(x + \sqrt{x^2 + y^2}).$$

[2]

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[4]

[illegible]

[illegible]

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