

Cambridge
International
A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

FURTHER MATHEMATICS

9231/12

Paper 1

May/June 2018

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **27** printed pages and **1** blank page.



Find the length of the arc of C from the point where $t = 0$ to the point where $t = 3$. [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3

- 2** It is given that $f(n) = 2^{3n} + 8^{n-1}$. By simplifying $f(k) + f(k+1)$, or otherwise, prove by mathematical induction that $f(n)$ is divisible by 9 for every positive integer n . [6]

[illegible]

4

3 The curve C has polar equation $r = \cos 2\theta$, for $-\frac{1}{4}\pi \leq \theta \leq \frac{1}{4}\pi$.

(i) Sketch C . [2]

(ii) Find the area of the region enclosed by C , showing full working. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Find a cartesian equation of C . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

$$x^3 - 21x^2 + kx - 216 = 0,$$

where k is a constant, has real roots a , ar and ar^{-1} .

- (i) Find the numerical values of the roots.

[6]

[illegible]

7

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Deduce the value of k .

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5 Let $S_n = \sum_{r=1}^n (-1)^{r-1} r^2$.

(i) Use the standard result for $\sum_{r=1}^n r^2$ given in the List of Formulae (MF10) to show that

$S_{2n} = -n(2n + 1).$ [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) State the value of $\lim_{n \rightarrow \infty} \frac{S_{2n}}{n^2}$ and find $\lim_{n \rightarrow \infty} \frac{S_{2n+1}}{n^2}$. [4]

[illegible]

6 The curve C has equation

$$y = \frac{x^2 + b}{x + b},$$

where b is a positive constant.

(i) Find the equations of the asymptotes of C . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Show that C does not intersect the x -axis. [1]

.....

.....

.....

.....

.....

(iii) Justifying your answer, find the number of stationary points on C . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iv) Sketch C . Your sketch should indicate the coordinates of any points of intersection with the y -axis. You do not need to find the coordinates of any stationary points. [3]

given that when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 0$. [10]

[illegible]

8 The linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ is represented by the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & \alpha & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix}$$

and α is a constant. When $\alpha \neq 0$ the null space of T is denoted by K_1 .

(i) Find a basis for K_1 . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

15

When $\alpha = 0$ the null space of T is denoted by K_2 .

(ii) Find a basis for K_2 .

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Determine, justifying your answer, whether K_1 is a subspace of K_2 .

[2]

.....

.....

.....

.....

.....

.....

.....

.....

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.
$$I_n = \int_0^{\frac{1}{4}\pi} \sec^n x \tan^2 x \, dx.$$

(ii) Using the result that $\frac{d}{dx}(\sec x) = \tan x \sec x$, show that, for $n \geq 2$,

$$(n+1)I_n = (\sqrt{2})^{n-2} + (n-2)I_{n-2}. \quad [5]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Hence find the mean value of $\sec^4 x \tan^2 x$ with respect to x over the interval $0 \leq x \leq \frac{1}{4}\pi$, giving your answer in exact form. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

10 The line l_1 is parallel to the vector $a\mathbf{i} - \mathbf{j} + \mathbf{k}$, where a is a constant, and passes through the point whose position vector is $9\mathbf{j} + 2\mathbf{k}$. The line l_2 is parallel to the vector $-a\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ and passes through the point whose position vector is $-6\mathbf{i} - 5\mathbf{j} + 10\mathbf{k}$.

(i) It is given that l_1 and l_2 intersect.

(a) Show that $a = -\frac{6}{13}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

[illegible]

11 Answer only **one** of the following two alternatives.

EITHER

(i) Show that if $z = e^{i\theta}$ and $z \neq -1$ then

$$\frac{z-1}{z+1} = i \tan \frac{1}{2} \theta. \quad [3]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

$$\frac{z^3-1}{z^3+1} + \frac{z^2-1}{z^2+1} + \frac{z-1}{z+1} = 0. \quad [5]$$

[illegible]

$$(z^3 - 1)(z^2 + 1)(z + 1) + (z^2 - 1)(z^3 + 1)(z + 1) + (z - 1)(z^3 + 1)(z^2 + 1) = 0$$

and find the other three roots. Give your answers in an exact form. [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

OR

It is given that **e** is an eigenvector of the matrix **A**, with corresponding eigenvalue λ .

- (i) Write down another eigenvector of **A** corresponding to λ . [1]

.....

.....

.....

- (ii) Write down an eigenvector and corresponding eigenvalue of $\mathbf{A}^n$, where n is a positive integer. [2]

.....

.....

.....

Let $\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 7 & 0 \\ 4 & 8 & 1 \end{pmatrix}$.

- (iii) Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^n = \mathbf{PDP}^{-1}$. [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

$$\sum_{n=1}^{\infty} k^n (\mathbf{A}^n - k \mathbf{A}^{n+1}) = k \mathbf{A}. \quad [4]$$

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge International Examinations Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cie.org.uk after the live examination series.

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.