

Cambridge  
International  
**A Level**

**Cambridge International Examinations**  
Cambridge International Advanced Level

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**FURTHER MATHEMATICS**

**9231/13**

Paper 1

**May/June 2018**

**3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

**DO NOT WRITE IN ANY BARCODES.**

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **27** printed pages and **1** blank page.



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3

- 1** The variables  $x$  and  $y$  are such that  $y = -1$  when  $x = 0$  and

$$\left(x + \frac{dy}{dx}\right)^3 = y^2 + x.$$

- (i) Find the value of  $\frac{dy}{dx}$  when  $x = 0$ . [1]

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- (ii) Find also the value of  $\frac{d^2y}{dx^2}$  when  $x = 0$ . [4]

[illegible]

**4****2** (i) Verify that

$$\frac{n(e-1)+e}{n(n+1)e^{n+1}} = \frac{1}{ne^n} - \frac{1}{(n+1)e^{n+1}}. \quad [1]$$

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Let  $S_N = \sum_{n=1}^N \frac{n(e-1)+e}{n(n+1)e^{n+1}}.$

(ii) Express  $S_N$  in terms of  $N$  and  $e$ . [2]

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This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

$$\cos 4\theta = \cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta. \quad [3]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

in the form  $\tan q\pi$ , where  $q$  is a positive rational number. [5]

[illegible]

**8****4** The curve  $C$  has equation

$$y = \frac{x^2 + 7x + 6}{x - 2}.$$

**(i)** Find the coordinates of the points of intersection of  $C$  with the axes. [2]

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**(ii)** Find the equation of each of the asymptotes of  $C$ . [3]

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**9****(iii)** Sketch *C*.**[3]**

5 It is given that **e** is an eigenvector of the matrix **A** with corresponding eigenvalue  $\lambda$ .

(i) Show that **e** is an eigenvector of  $\mathbf{A}^3$  and state the corresponding eigenvalue. [3]

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It is given that

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}.$$

(ii) Find a matrix **P** and a diagonal matrix **D** such that

$$\mathbf{A}^3 + \mathbf{I} = \mathbf{PDP}^{-1},$$

where **I** is the  $2 \times 2$  identity matrix. [5]

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[illegible]

6 The equation

$$9x^3 - 9x^2 + x - 2 = 0$$

has roots  $\alpha, \beta, \gamma$ .

(i) Use the substitution  $y = 3x - 1$  to show that  $3\alpha - 1, 3\beta - 1, 3\gamma - 1$  are the roots of the equation

$$y^3 - 2y - 7 = 0. \qquad [2]$$

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The sum  $(3\alpha - 1)^n + (3\beta - 1)^n + (3\gamma - 1)^n$  is denoted by  $S_n$ .

(ii) Find the value of  $S_3$ . [2]

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(iii) Find the value of  $S_{-2}$ .

[4]

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7 The lines  $l_1$  and  $l_2$  have vector equations

$\mathbf{r} = a\mathbf{i} + 9\mathbf{j} + 13\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$  and  $\mathbf{r} = -3\mathbf{i} + 7\mathbf{j} - 2\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$

respectively. It is given that  $l_1$  and  $l_2$  intersect.

(i) Find the value of the constant  $a$ . [3]

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The point  $P$  has position vector  $3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$ .

(ii) Find the perpendicular distance from  $P$  to the plane containing  $l_1$  and  $l_2$ . [4]

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**(iii)** Find the perpendicular distance from  $P$  to  $l_2$ . [4]

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8 The curves  $C_1$  and  $C_2$  have polar equations, for  $0 \leq \theta \leq \pi$ , as follows:

$$C_1: r = a,$$
$$C_2: r = 2a|\cos \theta|,$$

where  $a$  is a positive constant. The curves intersect at the points  $P_1$  and  $P_2$ .

(i) Find the polar coordinates of  $P_1$  and  $P_2$ .

[2]

(ii) In a single diagram, sketch  $C_1$ ,  $C_2$  and their line of symmetry.

[3]

- (iii) The region  $R$  enclosed by  $C_1$  and  $C_2$  is bounded by the arcs  $OP_1$ ,  $P_1P_2$  and  $P_2O$ , where  $O$  is the pole. Find the area of  $R$ , giving your answer in exact form. [5]

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9 For the sequence  $u_1, u_2, u_3, \dots$ , it is given that  $u_1 = 8$  and

$$u_{r+1} = \frac{5u_r - 3}{4}$$

for all  $r$ .

(i) Prove by mathematical induction that

$$u_n = 4\left(\frac{5}{4}\right)^n + 3,$$

for all positive integers  $n$ . [5]

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(ii) Deduce the set of values of  $x$  for which the infinite series

$$(u_1 - 3)x + (u_2 - 3)x^2 + \dots + (u_r - 3)x^r + \dots$$

is convergent.

[2]

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(iii) Use the result given in part (i) to find surds  $a$  and  $b$  such that

$$\sum_{n=1}^N \ln(u_n - 3) = N^2 \ln a + N \ln b. \quad [3]$$

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10 It is given that  $t \neq 0$  and

$$t \frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 9tx = 3t^2 + 1.$$

(i) Show that if  $y = tx$  then

$$\frac{d^2y}{dt^2} + 9y = 3t^2 + 1.$$

[3]

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(ii) Find  $x$  in terms of  $t$ , given that  $x = \frac{1}{9}\pi$  and  $\frac{dx}{dt} = \frac{2}{3}$  when  $t = \frac{1}{3}\pi$ . [9]

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This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

(i) Show that

$$\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} e^x \cos x \, dx = \frac{1}{2} \left( e^{\frac{1}{2}\pi} + e^{-\frac{1}{2}\pi} \right). \quad [4]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(ii) It is given that, for  $n \geq 0$ ,

$$I_n = \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} e^{2x} \cos^n x \, dx.$$

Show that, for  $n \geq 2$ ,

$$4I_n = n(n-1) \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} e^{2x} \sin^2 x \cos^{n-2} x \, dx - nI_n,$$

and deduce the reduction formula

$$(n^2 + 4)I_n = n(n-1)I_{n-2}. \tag{6}$$

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- (iii) Using the result in part (i) and the reduction formula in part (ii), find the y-coordinate of the centroid of the region bounded by the x-axis and the arc of the curve  $y = e^x \cos x$  from  $x = -\frac{1}{2}\pi$  to  $x = \frac{1}{2}\pi$ . Give your answer correct to 3 significant figures. [4]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced, horizontal dotted lines across the entire page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -2 \\ -5 \\ 5 \\ 6 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 0 \\ -3 \\ 15 \\ 18 \end{pmatrix} \quad \text{and} \quad \mathbf{v}_4 = \begin{pmatrix} 0 \\ -2 \\ 10 \\ 8 \end{pmatrix}.$$

- [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (ii) Express  $\mathbf{v}_4$  as a linear combination of  $\mathbf{v}_1$ ,  $\mathbf{v}_2$  and  $\mathbf{v}_3$ . [2]

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- (iii) Write down a basis for  $V$ . [1]

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$$\text{Let } \mathbf{M} = \begin{pmatrix} 1 & -2 & 0 & 0 \\ 2 & -5 & -3 & -2 \\ 0 & 5 & 15 & 10 \\ 2 & 6 & 18 & 8 \end{pmatrix}.$$

- (iv) Find the general solution of  $\mathbf{M}\mathbf{x} = \mathbf{v}_1 + \mathbf{v}_2$ . [6]

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The set of elements of  $\mathbb{R}^4$  which are not solutions of  $\mathbf{M}\mathbf{x} = \mathbf{v}_1 + \mathbf{v}_2$  is denoted by  $W$ .

(v) State, with a reason, whether  $W$  is a vector space. [2]

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Additional Page

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