

Cambridge
International
A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE
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FURTHER MATHEMATICS

9231/13

Paper 1

October/November 2018

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **27** printed pages and **1** blank page.



1 The vectors **a**, **b**, **c** and **d** in $\mathbb{R}^3$ are given by

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 9 \\ 0 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{d} = \begin{pmatrix} 0 \\ -8 \\ 3 \end{pmatrix}.$$

(i) Show that $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ is a basis for $\mathbb{R}^3$. [3]

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(ii) Express **d** in terms of **a**, **b** and **c**. [2]

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3**2** The roots of the equation

$$x^3 + px^2 + qx + r = 0$$

are α , 2α , 4α , where p , q , r and α are non-zero real constants.**(i)** Show that

$$2p\alpha + q = 0. \quad [4]$$

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(ii) Show that

$$p^3r - q^3 = 0. \quad [2]$$

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3 The sequence of positive numbers $u_1, u_2, u_3, \dots$ is such that $u_1 < 3$ and, for $n \geq 1$,

(i) By considering $3 - u_{n+1}$, or otherwise, prove by mathematical induction that $u_n < 3$ for all positive integers n . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[3]

[illegible]

4 A curve is defined parametrically by

$$x = t - \frac{1}{2} \sin 2t \quad \text{and} \quad y = \sin^2 t.$$

The arc of the curve joining the point where $t = 0$ to the point where $t = \pi$ is rotated through one complete revolution about the x -axis. The area of the surface generated is denoted by S .

(i) Show that

$$S = a\pi \int_0^\pi \sin^3 t \, dt,$$

where the constant a is to be found. [5]

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5 It is given that λ is an eigenvalue of the matrix $\mathbf{A}$ with $\mathbf{e}$ as a corresponding eigenvector, and μ is an eigenvalue of the matrix $\mathbf{B}$ for which $\mathbf{e}$ is also a corresponding eigenvector.

(i) Show that $\lambda + \mu$ is an eigenvalue of the matrix $\mathbf{A} + \mathbf{B}$ with $\mathbf{e}$ as a corresponding eigenvector. [2]

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The matrix $\mathbf{A}$, given by

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix}$$

has $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ as eigenvectors.

(ii) Find the corresponding eigenvalues. [3]

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The matrix **B** has eigenvalues 4, 5 and 1 with corresponding eigenvectors $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ respectively.

(iii) Find a matrix **P** and a diagonal matrix **D** such that $(\mathbf{A} + \mathbf{B})^3 = \mathbf{PDP}^{-1}$. [3]

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6 The curve C has equation

$$y = \frac{x^2 + ax - 1}{x + 1},$$

where a is constant and $a > 1$.

(i) Find the equations of the asymptotes of C . [3]

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(ii) Show that C intersects the x -axis twice. [1]

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(iii) Justifying your answer, find the number of stationary points on C . [2]

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(iv) Sketch C , stating the coordinates of its point of intersection with the y -axis. [3]

7 (i) Use de Moivre's theorem to show that

$$\sin 8\theta = 8 \sin \theta \cos \theta (1 - 10 \sin^2 \theta + 24 \sin^4 \theta - 16 \sin^6 \theta). \quad [6]$$

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(ii) Use the equation $\frac{\sin 8\theta}{\sin 2\theta} = 0$ to find the roots of

$$16x^6 - 24x^4 + 10x^2 - 1 = 0$$

in the form $\sin k\pi$, where k is rational. [4]

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8 The plane Π_1 has equation

$$\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}.$$

(i) Find a cartesian equation of Π_1 . [3]

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The plane Π_2 has equation $3x + y - z = 3$.

(ii) Find the acute angle between Π_1 and Π_2 , giving your answer in degrees. [2]

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(iii) Find an equation of the line of intersection of Π_1 and Π_2 , giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$.
[5]

[illegible]

9 The curve C has polar equation

$$r = 5\sqrt{(\cot \theta)},$$

where $0.01 \leq \theta \leq \frac{1}{2}\pi$.

- (i) Find the area of the finite region bounded by C and the line $\theta = 0.01$, showing full working. Give your answer correct to 1 decimal place. [3]

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Let P be the point on C where $\theta = 0.01$.

- (ii) Find the distance of P from the initial line, giving your answer correct to 1 decimal place. [2]

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[3]

[illegible]

(iv) Sketch C.

[2]

given that $x = 3$ and $\frac{dx}{dt} = 0$ when $t = 0$. [10]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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(ii) Show that, for large positive values of t and for any initial conditions,

$$x \approx \sqrt{37} \sin(3t - \phi),$$

where the constant ϕ is such that $\tan \phi = 6$. [3]

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(i) By considering $(2r+1)^2 - (2r-1)^2$, use the method of differences to prove that

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1). \quad [3]$$

[illegible]

21

- (ii)** By considering $(2r+1)^4 - (2r-1)^4$, use the method of differences and the result given in part (i) to prove that

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2. \quad [5]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

The sums S and T are defined as follows:

$$S = 1^3 + 2^3 + 3^3 + 4^3 + \dots + (2N)^3 + (2N + 1)^3,$$
$$T = 1^3 + 3^3 + 5^3 + 7^3 + \dots + (2N - 1)^3 + (2N + 1)^3.$$

(iii) Use the result given in part (ii) to show that $S = (2N + 1)^2(N + 1)^2$. [1]

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(iv) Hence, or otherwise, find an expression in terms of N for T , factorising your answer as far as possible. [2]

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(v) Deduce the value of $\frac{S}{T}$ as $N \rightarrow \infty$. [2]

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ORThe curve C has equation

$$x^2 + 2xy = y^3 - 2.$$

- (i) Show that $A(-1, 1)$ is the only point on C with x -coordinate equal to -1 . [2]

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For $n \geq 1$, let A_n denote the value of $\frac{d^n y}{dx^n}$ at the point $A(-1, 1)$.

- (ii) Show that $A_1 = 0$. [3]

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Let $I_n = \int_{-1}^0 x^n \frac{d^ny}{dx^n} \, dx.$

(iv) Show that for $n \geq 2,$

$$I_n = (-1)^{n+1}A_{n-1} - nI_{n-1}.$$

[3]

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(v) Deduce the value of I_3 in terms of I_1 . [2]

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This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

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