

Cambridge
International
A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

FURTHER MATHEMATICS

9231/11

Paper 1

October/November 2019

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **22** printed pages and **2** blank pages.



This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

2 It is given that $y = \ln(ax + 1)$, where a is a positive constant. Prove by mathematical induction that, for every positive integer n ,

$$\frac{d^n y}{dx^n} = (-1)^{n-1} \frac{(n-1)! a^n}{(ax+1)^n}. \quad [6]$$

[illegible]

4

3 The integral I_n , where n is a positive integer, is defined by

$$I_n = \int_{\frac{1}{2}}^1 x^{-n} \sin \pi x \, dx.$$

(i) Show that

$$n(n+1)I_{n+2} = 2^{n+1}n + \pi - \pi^2 I_n. \quad [5]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Find I_5 in terms of π and I_1 . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5

- 4** The line $y = 2x + 1$ is an asymptote of the curve C with equation

$$y = \frac{x^2 + 1}{ax + b}.$$

- (i)** Find the values of the constants a and b . [3]

.....

.....

.....

.....

.....

.....

.....

- (ii)** State the equation of the other asymptote of C . [1]

.....

.....

- (iii)** Sketch C . [Your sketch should indicate the coordinates of any points of intersection with the y -axis. You do not need to find the coordinates of any stationary points.] [3]

5 Let $S_N = \sum_{r=1}^N (5r+1)(5r+6)$ and $T_N = \sum_{r=1}^N \frac{1}{(5r+1)(5r+6)}$.

$$S_N = \frac{1}{3}N(25N^2 + 90N + 83). \quad [3]$$
This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Find $\lim_{N \rightarrow \infty} (N^{-3} S_N T_N)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

respectively.

- (i) Find the shortest distance between the lines OC and AB . [5]

[illegible]

[illegible]

7 The equation $x^3 + 2x^2 + x + 7 = 0$ has roots α, β, γ .

(i) Use the relation $x^2 = -7y$ to show that the equation

$$49y^3 + 14y^2 - 27y + 7 = 0$$

has roots $\frac{\alpha}{\beta\gamma}, \frac{\beta}{\gamma\alpha}, \frac{\gamma}{\alpha\beta}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Show that $\frac{\alpha^2}{\beta^2\gamma^2} + \frac{\beta^2}{\gamma^2\alpha^2} + \frac{\gamma^2}{\alpha^2\beta^2} = \frac{58}{49}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Find the exact value of $\frac{\alpha^3}{\beta^3\gamma^3} + \frac{\beta^3}{\gamma^3\alpha^3} + \frac{\gamma^3}{\alpha^3\beta^3}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

$$\mathbf{M} = \begin{pmatrix} 2 & m & 1 \\ 0 & m & 7 \\ 0 & 0 & 1 \end{pmatrix},$$

(i) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{M} = \mathbf{PDP}^{-1}$.

[7]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

13

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Find $\mathbf{M}^7\mathbf{P}$.**[3]**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

9 (i) Use de Moivre's theorem to show that

$$\sec 6\theta = \frac{\sec^6 \theta}{32 - 48 \sec^2 \theta + 18 \sec^4 \theta - \sec^6 \theta}. \quad [6]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(ii) Hence obtain the roots of the equation

$$3x^6 - 36x^4 + 96x^2 - 64 = 0$$

in the form $\sec q\pi$, where q is rational.

[5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

10 The matrix **A** is defined by

$$\mathbf{A} = \begin{pmatrix} 1 & 5 & 1 \\ 1 & -2 & -2 \\ 2 & 3 & \theta \end{pmatrix}.$$

(i) (a) Find the rank of **A** when $\theta \neq -1$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the rank of **A** when $\theta = -1$. [1]

.....

.....

.....

Consider the system of equations

$$\begin{aligned} x + 5y + z &= -1, \\ x - 2y - 2z &= 0, \\ 2x + 3y + \theta z &= \theta. \end{aligned}$$

(ii) Solve the system of equations when $\theta \neq -1$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Find the general solution when $\theta = -1$. [3]

.....

.....

.....

.....

.....

.....

.....

(iv) Show that if $\theta = -1$ and $\phi \neq -1$ then $\mathbf{Ax} = \begin{pmatrix} -1 \\ 0 \\ \phi \end{pmatrix}$ has no solution. [2]

.....

.....

.....

.....

.....

.....

.....

11 Answer only **one** of the following two alternatives.

EITHER

It is given that $w = \cos y$ and

$$\tan y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 2 \tan y \frac{dy}{dx} = 1 + e^{-2x} \sec y.$$

(i) Show that

$$\frac{d^2w}{dx^2} + 2 \frac{dw}{dx} + w = -e^{-2x}. \qquad [4]$$

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Find the particular solution for y in terms of x , given that when $x = 0$, $y = \frac{1}{3}\pi$ and $\frac{dy}{dx} = \frac{1}{\sqrt{3}}$. [10]

.....

.....

.....

.....

.....

.....

.....

.....

[illegible]

OR

The curves C_1 and C_2 have polar equations, for $0 \leq \theta \leq \frac{1}{2}\pi$, as follows:

$$\begin{aligned} C_1 : r &= 2(e^\theta + e^{-\theta}), \\ C_2 : r &= e^{2\theta} - e^{-2\theta}. \end{aligned}$$

The curves intersect at the point P where $\theta = \alpha$.

- (i) Show that $e^{2\alpha} - 2e^\alpha - 1 = 0$. Hence find the exact value of α and show that the value of r at P is $4\sqrt{2}$. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

21

(ii) Sketch C_1 and C_2 on the same diagram. [3]

(iii) Find the area of the region enclosed by C_1 , C_2 and the initial line, giving your answer correct to 3 significant figures. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[illegible]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.