



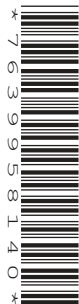
Cambridge International AS & A Level

CANDIDATE
NAME
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FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

2

1 Let a be a positive constant.

(a) Sketch the curve with equation $y = \frac{ax}{x+7}$. [2]

3

(b) Sketch the curve with equation $y = \left| \frac{ax}{x+7} \right|$ and find the set of values of x for which $\left| \frac{ax}{x+7} \right| > \frac{a}{2}$.
[4]

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2 The cubic equation $6x^3 + px^2 - 3x - 5 = 0$, where p is a constant, has roots α, β, γ .

(a) Find a cubic equation whose roots are $\alpha^2, \beta^2, \gamma^2$. [3]

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(b) It is given that $\alpha^2 + \beta^2 + \gamma^2 = 2(\alpha + \beta + \gamma)$.

(i) Find the value of p . [3]

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(ii) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. [2]

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3 The curve C has equation $y = \frac{x^2}{2x+1}$.

(a) Find the equations of the asymptotes of C . [3]

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(b) Find the coordinates of the stationary points on C . [3]

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(c) Sketch C .

[3]

where a and b are integers to be found.

[5]

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[illegible]

(c) Find the limit, as $n \rightarrow \infty$, of $\sum_{r=n+1}^{2n} \frac{n}{r^2 - 1}$. [4]

[illegible]

- 5** The lines l_1 and l_2 have equations $\mathbf{r} = 3\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 3\mathbf{i} - 5\mathbf{j} - 6\mathbf{k} + \mu(5\mathbf{j} + 6\mathbf{k})$ respectively.

- (a) Find the shortest distance between l_1 and l_2 . [5]

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11

The plane Π contains l_1 and is parallel to the vector $\mathbf{i} + \mathbf{k}$.

(b) Find the equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

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(c) Find the acute angle between l_2 and Π . [3]

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- Find the value of d . [3]

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- $$\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{pmatrix}. \quad [5]$$

This image shows a full page of handwriting practice paper. It features 18 horizontal rows, each defined by two parallel dashed lines. The lines are evenly spaced and extend across the entire width of the page, providing a guide for letter height and placement. There is no text or other markings on the page.

- (c) The line $y = 2x$ is invariant under the transformation in the x - y plane represented by $\mathbf{A}^n \mathbf{B}$, where
- $$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 33 & 0 \end{pmatrix}.$$

Find the value of n .

[5]

[illegible]

7 The curve C_1 has polar equation $r = \theta \cos \theta$, for $0 \leq \theta \leq \frac{1}{2}\pi$.

(a) The point on C_1 furthest from the line $\theta = \frac{1}{2}\pi$ is denoted by P . Show that, at P ,

$$2\theta \tan \theta - 1 = 0$$

and verify that this equation has a root between 0.6 and 0.7. [5]

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The curve C_2 has polar equation $r = \theta \sin \theta$, for $0 \leq \theta \leq \frac{1}{2}\pi$. The curves C_1 and C_2 intersect at the pole, denoted by O , and at another point Q .

(b) Find the polar coordinates of Q , giving your answers in exact form. [2]

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(c) Sketch C_1 and C_2 on the same diagram.

[3]

(d) Find, in terms of π , the area of the region bounded by the arc OQ of C_1 and the arc OQ of C_2 . [7]

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[illegible]

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.