



Cambridge International AS & A Level

CANDIDATE
NAME

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FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

May/June 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Blank pages are indicated.

	Grade awarded		
	A	B	C
Background music	49	51	40
Silence	93	68	49

Test at the 10% significance level whether grades awarded are independent of whether background music is playing during the examination. [6]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

2 The times, in milliseconds, taken by a computer to perform a certain task were recorded on 10 randomly chosen occasions. The times were as follows.

It is claimed that the median time to complete the task is 6.4 milliseconds.

- (a) Carry out a Wilcoxon signed-rank test at the 5% significance level to test this claim. [6]

[illegible]

- (b)** State an underlying assumption that is made when using a Wilcoxon signed-rank test. [1]

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$$f(x) = \begin{cases} \frac{3}{16}(2 - \sqrt{x}) & 0 \leq x < 1, \\ \frac{3}{16\sqrt{x}} & 1 \leq x \leq 9, \\ 0 & \text{otherwise.} \end{cases}$$

- [3]

[illegible]

(b) Find the probability density function of Y . [5]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal dashed lines spaced evenly across the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

The manager claims that there is no difference between the mean volume of coffee in cups filled by machine X and the mean volume of coffee in cups filled by machine Y .

[9]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(a) Assuming that distances are normally distributed, test, at the 5% significance level, whether the population mean distance thrown is more than 22.0 metres. [7]

[illegible]

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(b) Find a 95% confidence interval for the population mean distance thrown. [3]

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6 A bag contains 4 red balls and 6 blue balls. Rassa selects two balls at random, without replacement, from the bag. The number of red balls selected by Rassa is denoted by X .

(a) Find the probability generating function, $G_X(t)$, of X . [2]

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Rassa also tosses two coins. One coin is biased so that the probability of a head is $\frac{2}{3}$. The other coin is biased so that the probability of a head is p . The probability generating function of Y , the number of heads obtained by Rassa, is $G_Y(t)$. The coefficient of t in $G_Y(t)$ is $\frac{7}{12}$.

(b) Find $G_Y(t)$. [3]

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The random variable Z is the sum of the number of red balls selected and the number of heads obtained by Rassa.

(c) Find the probability generating function of Z , expressing your answer as a polynomial. [3]

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(d) Use the probability generating function of Z to find $E(Z)$. [2]

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This image shows a full page of a document template designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, and there are no margins, headers, or footers present.

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