



Cambridge International AS & A Level

CANDIDATE
NAME

CENTRE
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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

October/November 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.



This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) Deduce an approximation to $\int_0^{\frac{1}{5}} e^{-x^2} dx$, giving your answer as a rational fraction in its lowest terms. [2]

[illegible]

$$9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + y = 3x^2 + 30x.$$

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3 (a) Show that the system of equations

$$\begin{aligned}x - 2y - 4z &= 1, \\ x - 2y + kz &= 1, \\ -x + 2y + 2z &= 1,\end{aligned}$$

where k is a constant, does not have a unique solution. [2]

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(b) Given that $k = -4$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]

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- (c) Given instead that $k = -2$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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- (d) For the case where $k \neq -2$ and $k \neq -4$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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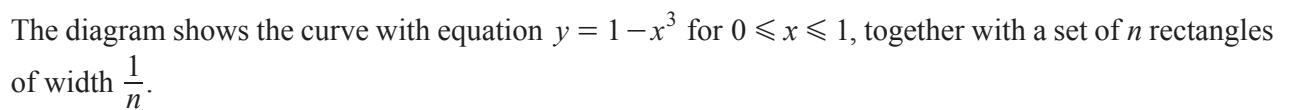
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$$\int_0^1 (1-x^3) dx \leq \frac{3n^2+2n-1}{4n^2}. \quad [4]$$
[illegible]

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5 It is given that

$$x = \sinh^{-1} t, \quad y = \cos^{-1} t,$$

where $-1 < t < 1$.

(a) By differentiating $\cos y$ with respect to t , show that $\frac{dy}{dt} = -\frac{1}{\sqrt{1-t^2}}$. [4]

[illegible]

[5]

[illegible]

[illegible]

for which $y = 0$ when $\theta = \frac{1}{2}\pi$. [6]

[illegible]

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

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respectively.

- [4]

[illegible]

$$\coth^2 x - \operatorname{cosech}^2 x = 1. \quad [3]$$
[illegible]

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- Show that $\cosh a = 2$ and find, in logarithmic form, the exact value of a . [7]

This image shows a full page of a worksheet designed for handwriting practice. It features 18 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, providing a clear guide for letter height and placement. There are no margins, text, or other markings present.

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, which is otherwise blank.

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