



Cambridge International AS & A Level

CANDIDATE
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FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

1 Find the Maclaurin’s series for $\tan\left(x + \frac{1}{4}\pi\right)$ up to and including the term in x^2 . [5]

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2 A curve has equation $y = \cosh x$, for $0 \leq x \leq \frac{1}{2}$.
Find, in terms of π and e, the area of the surface generated when the curve is rotated through 2π radians about the x -axis. [6]

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3 Find all the roots of the equation $(w + 1)^6 = 1$, giving your answers in the form $x + iy$ where x and y are real and exact. [4]

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$$x \frac{dy}{dx} + 2y = e^x$$

for which $y = 3$ when $x = 1$. Give your answer in the form $y = f(x)$.

[8]

[illegible]

5

5 The curve C has equation

$$y^2 + (xy + 1)^2 = 5.$$

(a) Show that, at the point $(1, 1)$ on C , $\frac{dy}{dx} = -\frac{2}{3}$. [3]

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(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(1, 1)$. [5]

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6 Find the particular solution of the differential equation

given that, when $t = 0$, $x = 1$ and $\frac{dx}{dt} = 0$. [11]

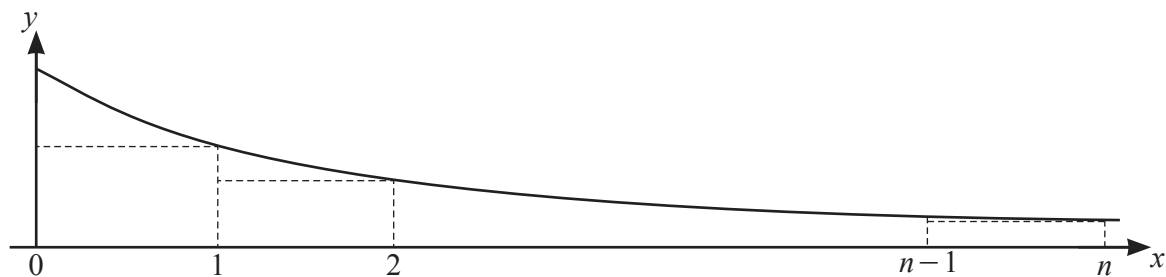
This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]

$$1 + 2 \sum_{r=1}^n \cos(2r\theta) = \frac{\sin(2n+1)\theta}{\sin \theta}. \quad [5]$$

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8



The diagram shows the curve $y = \frac{1}{\sqrt{x^2 + x + 1}}$ for $x \geq 0$, together with a set of n rectangles of unit width. By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{r^2 + r + 1}} < \ln\left(\frac{1}{3} + \frac{2}{3}n + \frac{2}{3}\sqrt{n^2 + n + 1}\right). \quad [10]$$

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$$ax + (2a + 5)y + (a + 1)z = 1,$$

$$-4y = 2,$$

$$3y - z = 3,$$

has a unique solution and interpret this situation geometrically. [3]

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13

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} a & 2a+5 & a+1 \\ 0 & -4 & 0 \\ 0 & 3 & -1 \end{pmatrix}.$$

- (b) Show that the eigenvalues of $\mathbf{A}$ are a , -1 and -4 . [2]

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- (c) Find a matrix $\mathbf{P}$ such that

$$\mathbf{A} = \mathbf{P} \begin{pmatrix} a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix} \mathbf{P}^{-1}. \quad [5]$$

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[illegible]

[6]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

This image shows a full page of a worksheet designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, and there are no margins, text, or other markings present.

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