



Cambridge International AS & A Level

CANDIDATE
NAME

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FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

October/November 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

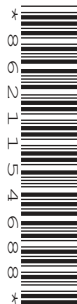
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Blank pages are indicated.



Find a 90% confidence interval for the difference between the population mean lengths of leaves in forests X and Y . [7]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 2 Metal rods produced by a certain factory are claimed to have a median breaking strength of 200 tonnes. For a random sample of 9 rods, the breaking strengths, measured in tonnes, were as follows.

210 186 188 208 184 191 215 198 196

A scientist believes that the median breaking strength of metal rods produced by this factory is less than 200 tonnes.

- (a) Use a Wilcoxon signed-rank test, at the 5% significance level, to test whether there is evidence to support the scientist’s belief. [6]

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- (b) Give a reason why a Wilcoxon signed-rank test is preferable to a sign test, when both are valid. [1]

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x	0	1	2	3	4	5
Frequency	12	39	46	37	12	4

[illegible]

[illegible]

Athlete	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>
Time before	48.8	48.2	50.3	49.6	49.4	48.9	47.6	50.3	48.4
Time after	47.9	47.8	49.6	49.1	49.6	48.9	47.7	49.1	48.1

Test at the 10% significance level whether the organiser's claim is justified, stating any assumption that you make. [8]

[illegible]

5 Keira has two unbiased coins. She tosses both coins. The number of heads obtained by Keira is denoted by X .

(a) Find the probability generating function $G_X(t)$ of X . [1]

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Hassan has three coins, two of which are biased so that the probability of obtaining a head when the coin is tossed is $\frac{1}{3}$. The corresponding probability for the third coin is $\frac{1}{4}$. The number of heads obtained by Hassan when he tosses these three coins is denoted by Y .

(b) Find the probability generating function $G_Y(t)$ of Y . [3]

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The random variable Z is the total number of heads obtained by Keira and Hassan.

(c) Find the probability generating function of Z , expressing your answer as a polynomial. [3]

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(d) Use the probability generating function of Z to find $E(Z)$. [2]

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(e) Use the probability generating function of Z to find the most probable value of Z . [1]

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6 The continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{60}(16x - x^2) & 0 \leq x \leq 6, \\ 1 & x > 6. \end{cases}$$

(a) Find the interquartile range of X . [4]

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(b) Find $E(X^3)$. [4]

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(c) Find the probability density function of Y . [3]

[illegible]

[illegible]

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