



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



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This image shows a full page of white paper designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines running from left to right across the entire width of the page. There are no margins, text, or other markings present.

- 2 (a) Use standard results from the List of formulae (MF19) to find $\sum_{r=1}^n (1-r-r^2)$ in terms of n , simplifying your answer. [3]

[illegible]

and hence use the method of differences to find $\sum_{r=1}^n \frac{1-r-r^2}{(r^2+2r+2)(r^2+1)}$. [5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1-r-r^2}{(r^2+2r+2)(r^2+1)}$. [1]

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- [illegible]

(b) Find the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3} + \frac{1}{\delta^3}$. [3]

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(c) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

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4 The matrix **M** represents the sequence of two transformations in the x - y plane given by a rotation of 60° anticlockwise about the origin followed by a one-way stretch in the x -direction, scale factor d ($d \neq 0$).

(a) Find **M** in terms of d . [4]

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(b) The unit square in the x - y plane is transformed by **M** onto a parallelogram of area $\frac{1}{2}d^2$ units².
Show that $d = 2$. [2]

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The matrix **N** is such that $\mathbf{MN} = \begin{pmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

(c) Find **N**. [3]

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(d) Find the equations of the invariant lines, through the origin, of the transformation in the *x-y* plane represented by **MN**. [5]

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5 The curve C has polar equation $r = a \cot\left(\frac{1}{3}\pi - \theta\right)$, where a is a positive constant and $0 \leq \theta \leq \frac{1}{6}\pi$.

(a) Sketch C and show that $a = 2$. [3]

(b) Find the exact value of the area of the region bounded by C , the initial line and the half-line $\theta = \frac{1}{6}\pi$. [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

6 Let t be a positive constant.

The line l_1 passes through the point with position vector $t\mathbf{i} + \mathbf{j}$ and is parallel to the vector $-2\mathbf{i} - \mathbf{j}$. The line l_2 passes through the point with position vector $\mathbf{j} + t\mathbf{k}$ and is parallel to the vector $-2\mathbf{j} + \mathbf{k}$.

It is given that the shortest distance between the lines l_1 and l_2 is $\sqrt{21}$.

(a) Find the value of t . [5]

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The plane Π_1 contains l_1 and is parallel to l_2 .

(b) Write down an equation of Π_1 , giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. [1]

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The plane Π_2 has Cartesian equation $5x - 6y + 7z = 0$.

(c) Find the acute angle between l_2 and Π_2 . [3]

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(d) Find the acute angle between Π_1 and Π_2 . [3]

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7 The curve C has equation $y = \frac{x^2 + x + 9}{x + 1}$.

(a) Find the equations of the asymptotes of C . [3]

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(b) Find the coordinates of the stationary points on C . [4]

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- (c) Sketch C , stating the coordinates of any intersections with the axes. [3]

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- (d) Sketch the curve with equation $y = \left| \frac{x^2 + x + 9}{x + 1} \right|$ and find the set of values of x for which $2|x^2 + x + 9| > 13|x + 1|$. [5]

[illegible]

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