



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

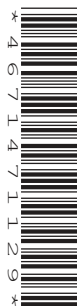
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



1 (a) Given that a is an integer, show that the system of equations

$$\begin{aligned}ax + 3y + z &= 14, \\ 2x + y + 3z &= 0, \\ -x + 2y - 5z &= 17,\end{aligned}$$

has a unique solution and interpret this situation geometrically. [4]

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(b) Find the value of a for which $x = 1$, $y = 4$, $z = -2$ is the solution to the system of equations in part (a). [1]

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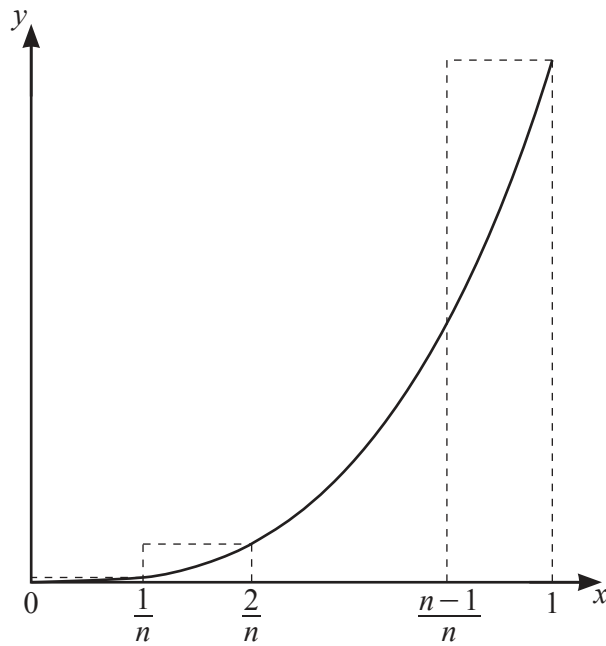
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(a) Find the general solution for y in terms of x . [6]

(b) State an approximate solution for large positive values of x . [1]

[Turn over

3



The diagram shows the curve with equation $y = x^3$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 x^3 dx < U_n$, where

$$U_n = \left(\frac{n+1}{2n}\right)^2. \quad [4]$$

[illegible]

(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 x^3 \mathrm{d}x$. [4]

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(c) Find the least value of n such that $U_n - L_n < 10^{-3}$. [2]

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where $0 < \theta < \pi$, given that $y = 1$ when $\theta = \frac{1}{2}\pi$. Give your answer in the form $y = f(\theta)$. [9]

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(b) Given that z is an n th root of unity and $z \neq 1$, deduce that $1 + z + z^2 + \dots + z^{n-1} = 0$. [2]

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(c) Given instead that $z = \frac{1}{3}(\cos \theta + i \sin \theta)$, use de Moivre's theorem to show that

$$\sum_{m=1}^{\infty} 3^{-m} \cos m\theta = \frac{3 \cos \theta - 1}{10 - 6 \cos \theta} . \quad [7]$$

[illegible]

$$\mathbf{A} = \begin{pmatrix} 5 & -\frac{22}{3} & 8 \\ 0 & -6 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- (a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^2 = \mathbf{PDP}^{-1}$. [7]

This image shows a full page of blank handwriting practice paper. It features multiple sets of horizontal lines across the entire page. Each set consists of three lines: a solid top line, a dashed middle line, and a solid bottom line. These sets are repeated vertically down the page, providing a guide for letter height and placement. The background is white, and the lines are printed in a light gray or blue color. There is no text or other markings on the page.

[illegible]

7 (a) It is given that $y = \operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$.

Express $\cosh y$ in terms of x and hence show that $\sinh y \frac{dy}{dx} = -\frac{1}{\left(x + \frac{1}{2}\right)^2}$. [3]

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(b) Find the first three terms in the Maclaurin’s series for $\operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$ in the form

$$\ln a + bx + cx^2,$$

where a , b and c are constants to be determined. [7]

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$$x = 2 \cosh t, \quad y = \frac{3}{2}t - \frac{1}{4} \sinh 2t, \quad \text{for } 0 \leq t \leq 1.$$

- (a) Find $\frac{dx}{dt}$ and show that $\frac{dy}{dt} = 1 - \sinh^2 t$. [3]

The area of the surface generated when C is rotated through 2π radians about the x -axis is denoted by A .

- (b) (i)** Show that $A = \pi \int_0^1 \left(\frac{3}{2}t - \frac{1}{4} \sinh 2t \right) (1 + \cosh 2t) dt$. [4]

[illegible]

[6]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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