



# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

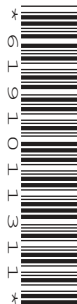
## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.



1 (a) Given that  $a$  is an integer, show that the system of equations

$$\begin{aligned}ax + 3y + z &= 14, \\ 2x + y + 3z &= 0, \\ -x + 2y - 5z &= 17,\end{aligned}$$

has a unique solution and interpret this situation geometrically. [4]

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(b) Find the value of  $a$  for which  $x = 1$ ,  $y = 4$ ,  $z = -2$  is the solution to the system of equations in part (a). [1]

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- 2** The variables  $x$  and  $y$  are related by the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2x + 1.$$

- (a) Find the general solution for  $y$  in terms of  $x$ . [6]

[illegible]

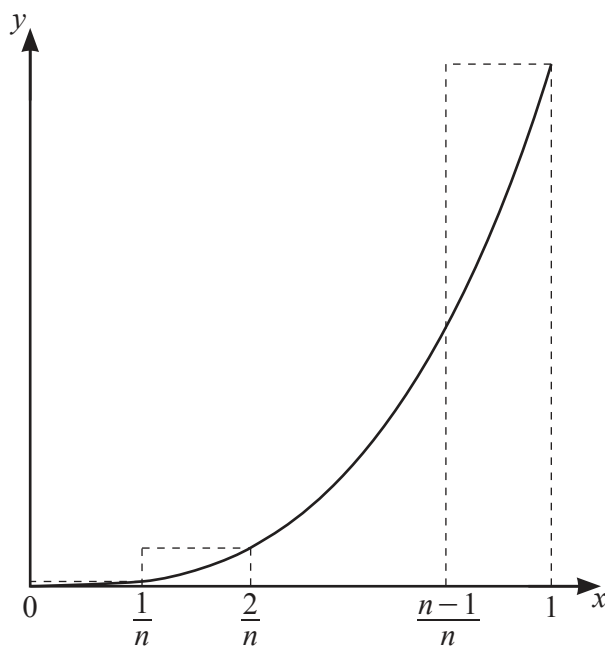
- (b) State an approximate solution for large positive values of  $x$ . [1]

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The diagram shows the curve with equation  $y = x^3$  for  $0 \leq x \leq 1$ , together with a set of  $n$  rectangles of width  $\frac{1}{n}$ .

(a) By considering the sum of the areas of these rectangles, show that  $\int_0^1 x^3 dx < U_n$ , where

$$U_n = \left(\frac{n+1}{2n}\right)^2. \quad [4]$$

[illegible]

(b) Use a similar method to find, in terms of  $n$ , a lower bound  $L_n$  for  $\int_0^1 x^3 \mathrm{d}x$ . [4]

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(c) Find the least value of  $n$  such that  $U_n - L_n < 10^{-3}$ . [2]

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where  $0 < \theta < \pi$ , given that  $y = 1$  when  $\theta = \frac{1}{2}\pi$ . Give your answer in the form  $y = f(\theta)$ . [9]

[illegible]



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$$\sum_{m=1}^{\infty} 3^{-m} \cos m\theta = \frac{3 \cos \theta - 1}{10 - 6 \cos \theta}. \quad [7]$$
[illegible]

[illegible]

$$\mathbf{A} = \begin{pmatrix} 5 & -\frac{22}{3} & 8 \\ 0 & -6 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- [illegible]

[illegible]

7 (a) It is given that  $y = \operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$ .

Express  $\cosh y$  in terms of  $x$  and hence show that  $\sinh y \frac{dy}{dx} = -\frac{1}{\left(x + \frac{1}{2}\right)^2}$ . [3]

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(b) Find the first three terms in the Maclaurin’s series for  $\operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$  in the form

$$\ln a + bx + cx^2,$$

where  $a$ ,  $b$  and  $c$  are constants to be determined. [7]

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$$x = 2 \cosh t, \quad y = \frac{3}{2}t - \frac{1}{4} \sinh 2t, \quad \text{for } 0 \leq t \leq 1.$$

- (a) Find  $\frac{dx}{dt}$  and show that  $\frac{dy}{dt} = 1 - \sinh^2 t$ . [3]

[illegible]

The area of the surface generated when  $C$  is rotated through  $2\pi$  radians about the  $x$ -axis is denoted by  $A$ .

- (b) (i)** Show that  $A = \pi \int_0^1 \left( \frac{3}{2}t - \frac{1}{4} \sinh 2t \right) (1 + \cosh 2t) dt$ . [4]

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[6]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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