



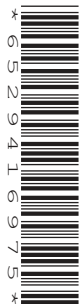
Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

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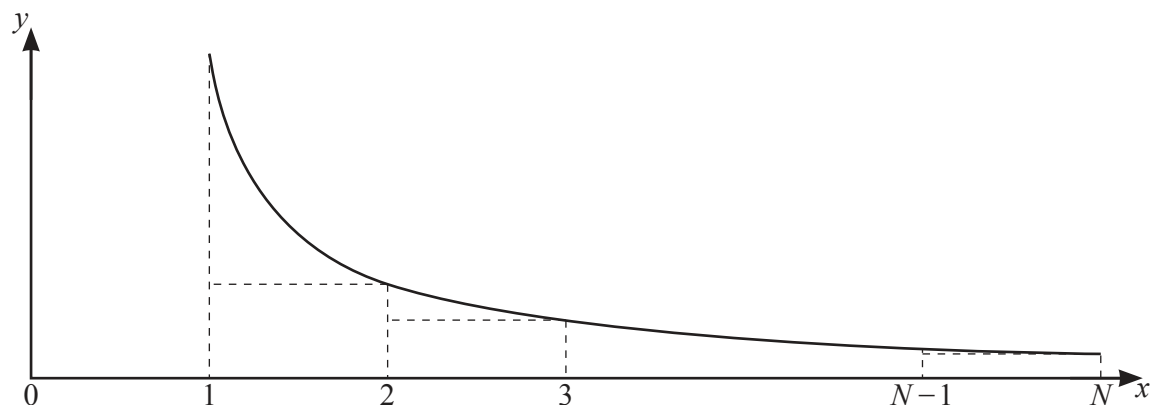
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$$z^8 - iz^5 - z^3 + i = 0,$$

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

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3



The diagram shows the curve $y = \frac{x}{2x^2 - 1}$ for $x \geq 1$, together with a set of $N-1$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^N \frac{r}{2r^2-1} < \frac{1}{4} \ln(2N^2-1) + 1. \quad [7]$$

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

$$\tan^5 \theta = \frac{\sin 5\theta - a \sin 3\theta + b \sin \theta}{\cos 5\theta + a \cos 3\theta + b \cos \theta},$$

[7]

[illegible]

5 The variables x and y are related by the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 4e^{-x}.$$

- (a) Find the value of the constant k such that $y = kxe^{-x}$ is a particular integral of the differential equation. [4]

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- (b) Find the solution of the differential equation for which $y = \frac{dy}{dx} = \frac{1}{2}$ when $x = 0$. [6]

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This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

This image shows a full page of white paper with ten evenly spaced horizontal dashed lines, typical of primary school handwriting practice paper. The lines extend across the entire width of the page.

- $$\frac{dy}{dx} + y \coth x = 4 \sinh x$$

[illegible]

7 The integral I_n , where n is an integer, is defined by $I_n = \int_0^{\frac{3}{2}} (4+x^2)^{-\frac{1}{2}n} \, dx$.

(a) Find the exact value of I_1 , expressing your answer in logarithmic form. [3]

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(b) By considering $\frac{d}{dx} \left(x(4+x^2)^{-\frac{1}{2}n} \right)$, or otherwise, show that

$$4nI_{n+2} = \frac{3}{2} \left(\frac{2}{5} \right)^n + (n-1)I_n. \qquad [5]$$

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(c) Find the value of I_5 . [3]

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$$2x + 6y - 5z = 0,$$

[2]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 13 & 18 & -28 \\ -4 & -1 & 8 \\ 2 & 6 & -5 \end{pmatrix}.$$

(b) Find the eigenvalue of $\mathbf{A}$ corresponding to the eigenvector $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$. [1]

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} = \mathbf{PDP}^{-1}$. [8]

[illegible]

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[illegible]

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