



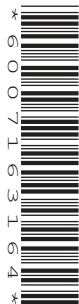
Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/33

Paper 3 Further Mechanics

May/June 2021

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

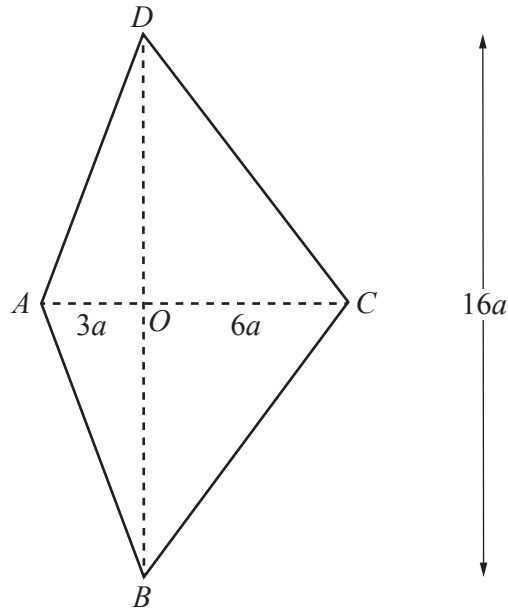
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 ms^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

1



A uniform lamina $ABCD$ consists of two isosceles triangles ABD and BCD . The diagonals of $ABCD$ meet at the point O . The length of AO is $3a$, the length of OC is $6a$ and the length of BD is $16a$ (see diagram).

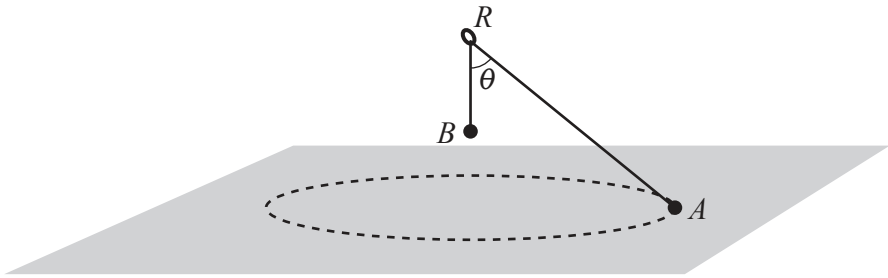
Find the distance of the centre of mass of the lamina from DB .

[3]

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[illegible]

3



Particles A and B , of masses $3m$ and m respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R . Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles on a smooth horizontal surface with speed $\frac{2}{5}\sqrt{ga}$. The angle between AR and BR is θ (see diagram). The normal reaction between A and the surface is $\frac{12}{5}mg$.

(a) Find $\cos \theta$. [3]

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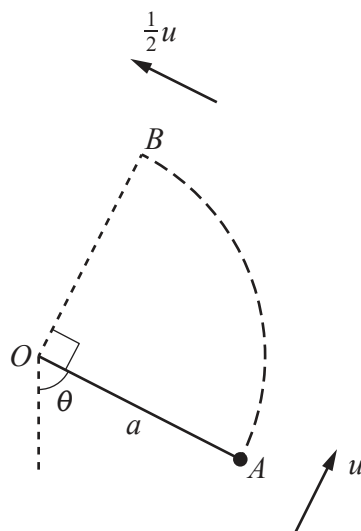
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4



A particle of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle is initially held with the string taut at the point A , where OA makes an angle θ with the downward vertical through O . The particle is then projected with speed u perpendicular to OA and begins to move upwards in part of a vertical circle. The string goes slack when the particle is at the point B where angle AOB is a right angle. The speed of the particle when it is at B is $\frac{1}{2}u$ (see diagram).

Find the tension in the string at A , giving your answer in terms of m and g .

[8]

[illegible]

- [illegible]

The displacement of P from O is x m at time t s.

- (b) Find an expression for x in terms of t , while P is moving upwards. [2]

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- (c) Find, correct to 3 significant figures, the greatest height above O reached by P . [2]

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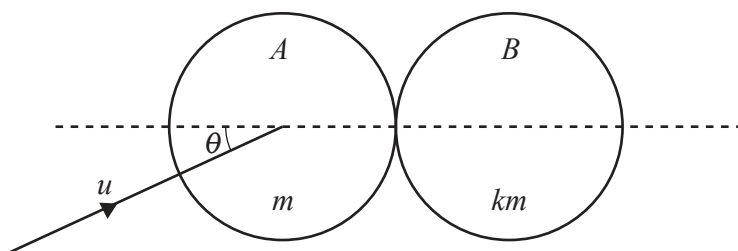
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6



Two uniform smooth spheres A and B of equal radii have masses m and km respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides with sphere B which is at rest. Immediately before the collision, A 's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

- (a) Show that the speed of B after the collision is $\frac{4u \cos \theta}{3(1+k)}$. [3]

[illegible]

[6]

[illegible]

7 A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

- (a) Use the equation of the trajectory given in the List of formulae (MF19), together with the condition $y = 0$, to establish an expression for the range R in terms of u , θ and g . [2]

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- (b) Deduce an expression for the maximum height H , in terms of u , θ and g . [2]

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It is given that $R = \frac{4H}{\sqrt{3}}$.

- (c) Show that $\theta = 60^\circ$. [1]

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It is given also that $u = \sqrt{40} \text{ m s}^{-1}$.

- (d) Find, by differentiating the equation of the trajectory or otherwise, the set of values of x for which the direction of motion makes an angle of less than 45° with the horizontal. [4]

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