



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

May/June 2021

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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1 A random sample of 7 observations of a variable X are as follows.

8.26 7.78 7.92 8.04 8.27 7.95 8.34

The population mean of X is μ .

(a) Test, at the 10% significance level, the null hypothesis $\mu = 8.22$ against the alternative hypothesis $\mu < 8.22$. [6]

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(b) State an assumption necessary for the test in part (a) to be valid. [1]

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	Instructor <i>A</i>	Instructor <i>B</i>	Instructor <i>C</i>	Instructor <i>D</i>	Total
Pass	72	42	52	68	234
Fail	33	34	41	58	166
Total	105	76	93	126	400

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

3 The continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{81}x^2 & 0 \leq x \leq 9, \\ 1 & x > 9. \end{cases}$$

(a) Find $E(\sqrt{X})$. [3]

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(b) Find $\text{Var}(\sqrt{X})$. [2]

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[illegible]

- 4 A scientist is investigating the lengths of the leaves of birch trees in different regions. He takes a random sample of 50 leaves from birch trees in region A and a random sample of 60 leaves from birch trees in region B . He records their lengths in cm, x and y , respectively. His results are summarised as follows.

$$\Sigma x = 282 \quad \Sigma x^2 = 1596 \quad \Sigma y = 328 \quad \Sigma y^2 = 1808$$

The population mean lengths of leaves from birch trees in regions A and B are μ_A cm and μ_B cm respectively.

Carry out a test at the 5% significance level to test the null hypothesis $\mu_A = \mu_B$ against the alternative hypothesis $\mu_A \neq \mu_B$. [8]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Employee	A	B	C	D	E	F	G	H	I	J	K
Uniform X	82	74	42	59	60	73	94	98	62	36	50
Uniform Y	78	75	63	56	67	82	99	90	72	48	61

- [illegible]

- [illegible]

- 6 Tanji has a bag containing 4 red balls and 2 blue balls. He selects 3 balls at random from the bag, without replacement. The number of red balls selected by Tanji is denoted by X .

(a) Find the probability generating function $G_X(t)$ of X . [2]

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Tanji also has two coins, each biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$. He throws the two coins at the same time. The number of heads obtained is denoted by Y .

(b) Find the probability generating function $G_Y(t)$ of Y . [2]

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The random variable Z is the sum of the number of red balls selected by Tanji and the number of heads obtained.

(c) Find the probability generating function of Z , expressing your answer as a polynomial. [3]

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[illegible]

- (d) Use the probability generating function of Z to find $E(Z)$ and $\text{Var}(Z)$. [5]

[illegible]

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