



Cambridge Assessment
International Education

Cambridge International AS & A Level

CANDIDATE
NAME

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FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

$$\alpha + \beta + \gamma = 3, \quad \alpha^2 + \beta^2 + \gamma^2 = 5, \quad \alpha^3 + \beta^3 + \gamma^3 = 6.$$

The cubic equation $x^3 + bx^2 + cx + d = 0$ has roots α, β, γ .

Find the values of b , c and d .

[6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

- 2 (a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^n r(r+1)(r+2)$ in terms of n , fully factorising your answer. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]

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3 The sequence of real numbers $a_1, a_2, a_3, \dots$ is such that $a_1 = 1$ and

$$a_{n+1} = \left(a_n + \frac{1}{a_n}\right)^3.$$

(a) Prove by mathematical induction that $\ln a_n \geq 3^{n-1} \ln 2$ for all integers $n \geq 2$. [6]

[You may use the fact that $\ln\left(x + \frac{1}{x}\right) > \ln x$ for $x > 0$.]

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(b) Show that $\ln a_{n+1} - \ln a_n > 3^{n-1} \ln 4$ for $n \geq 2$. [2]

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- State the type of each transformation, and make clear the order in which they are applied. [2]

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- This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, which is otherwise blank.

5 The plane Π has equation $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} + \mathbf{k}) + \mu(2\mathbf{i} + 3\mathbf{j})$.

(a) Find a Cartesian equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

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The line l passes through the point P with position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and is parallel to the vector $\mathbf{k}$.

(b) Find the position vector of the point where l meets Π . [3]

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(c) Find the acute angle between l and Π .

[3]

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(d) Find the perpendicular distance from P to Π .

[3]

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- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[2]

(c) Find the area of the region bounded by C and the initial line, giving your answer in exact form. [6]

This image shows a full page of a document template designed for handwriting practice. It features 18 horizontal rows. Each row is defined by three parallel lines: a solid top line, a dashed middle line, and a solid bottom line. The lines are evenly spaced and extend across the entire width of the page, providing a guide for letter height and placement. There is no text or other content on the page.

7 The curve C has equation $y = \frac{4x+5}{4-4x^2}$.

(a) Find the equations of the asymptotes of C . [2]

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(b) Find the coordinates of any stationary points on C . [4]

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13**(c)** Sketch C , stating the coordinates of the intersections with the axes.

[3]

(d) Sketch the curve with equation $y = \left| \frac{4x+5}{4-4x^2} \right|$ and find in exact form the set of values of x for which $4|4x+5| > 5|4-4x^2|$.

[6]

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[illegible]

Additional Page

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