



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

- 1 (a) Give full details of the geometrical transformation in the x - y plane represented by the matrix $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$. [1]

.....

.....

Let $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & 2 \end{pmatrix}$.

- (b) The triangle DEF in the x - y plane is transformed by $\mathbf{A}$ onto triangle PQR .
Given that the area of triangle DEF is 13 cm^2 , find the area of triangle PQR . [2]

.....

.....

- (c) Find the matrix $\mathbf{B}$ such that $\mathbf{AB} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$. [2]

.....

.....

.....

.....

.....

- (d) Show that the origin is the only invariant point of the transformation in the x - y plane represented by $\mathbf{A}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

$$\frac{d^n y}{dx^n} = (a^n x + n a^{n-1}) e^{ax}. \quad [6]$$
This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.

(a) Using the method of differences, or otherwise, show that $S_n = \ln \frac{n+2}{2(n+1)}$. [4]

[illegible]

Let $S = \sum_{r=1}^{\infty} \ln \frac{r(r+2)}{(r+1)^2}$.

(b) Find the least value of n such that $S_n - S < 0.01$. [3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

4 The cubic equation $x^3 + 2x^2 + 3x + 3 = 0$ has roots α, β, γ .

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

.....

.....

.....

.....

.....

.....

.....

(b) Show that $\alpha^3 + \beta^3 + \gamma^3 = 1$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

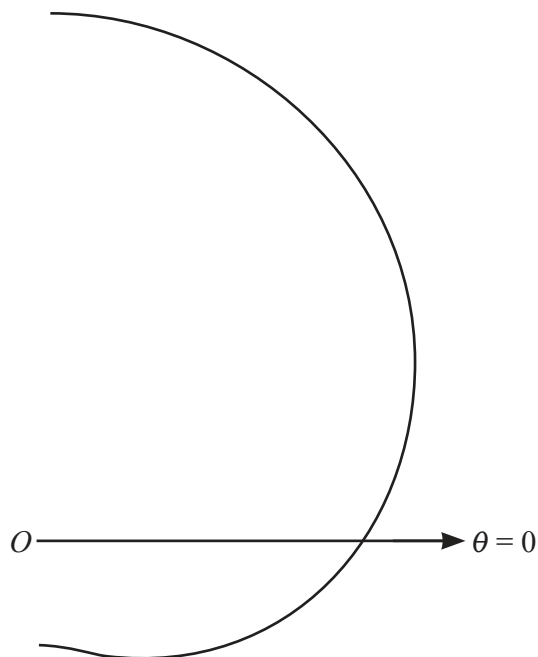
.....

where a , b and c are constants to be determined.

[6]

[illegible]

- [1]



(b) Add l to the diagram in part (a) and find the polar coordinates of the points of intersection of C and l . [5]

[illegible]

- Find the area of R , giving your answer in exact form. [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]

This image shows a full page of a worksheet designed for handwriting practice. It features 18 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, providing a clear guide for letter formation and alignment. There are no margins, text, or other markings present.

11

(c) Sketch C .

[2]

- (d) (i) Sketch the graphs of $y = \left| \frac{x^2}{x-3} \right|$ and $y = |x| - 3$ on a single diagram, stating the coordinates of the intersections with the axes. [4]

- (ii) Use your sketch to find the set of values of c for which $\left| \frac{x^2}{x-3} \right| \leq |x| + c$ has no solution. [1]

.....
.....

7 The points A, B, C have position vectors

$2\mathbf{i} + 2\mathbf{j}, \quad -\mathbf{j} + \mathbf{k} \quad \text{and} \quad 2\mathbf{i} + \mathbf{j} - 7\mathbf{k}$

respectively, relative to the origin O .

(a) Find an equation of the plane OAB , giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = p$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The plane Π has equation $x - 3y - 2z = 1$.

(b) Find the perpendicular distance of Π from the origin. [1]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the acute angle between the planes OAB and Π . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) Find an equation for the common perpendicular to the lines OC and AB . [10]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Additional Page

[illegible]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.