



**Cambridge Assessment
International Education**

Cambridge International AS & A Level

CANDIDATE
NAME

CENTRE
NUMBER

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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



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$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Use the characteristic equation of $\mathbf{A}$ to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I},$$

where p and q are integers to be determined.

[6]

[illegible]

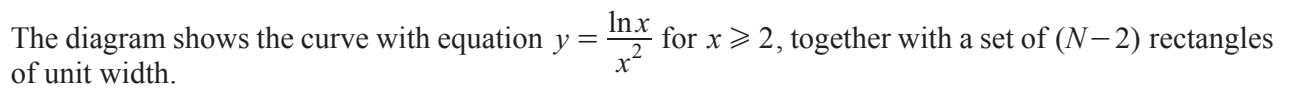
$$xy^3 - 4x^3y = 3.$$

- (a)** Show that, at the point $(-1, 1)$ on C , $\frac{dy}{dx} = 11$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[5]

This image shows a full page of white paper designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines running across the width of the page. There are no margins, text, or other markings present.


$$\sum_{r=1}^N \frac{\ln r}{r^2} < \frac{2+3 \ln 2}{4} - \frac{1+\ln N}{N}. \quad [7]$$
This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Use a similar method to find, in terms of N , a lower bound for $\sum_{r=1}^N \frac{\ln r}{r^2}$. [3]

[illegible]

given that, when $x = 0$, $y = -4$ and $\frac{dy}{dx} = 3$. [11]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

$$x^5 - 10x^4 + 40x^2 - 32 = 0$$

in the form $\operatorname{cosec}(q\pi)$, where q is rational. [4]

[illegible]

7 (a) Show that an appropriate integrating factor for

$$\sqrt{x^2-1} \frac{dy}{dx} + y = x^2 - x\sqrt{x^2-1}$$

is $x + \sqrt{x^2 - 1}$.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

for which $y = 1$ when $x = \frac{5}{4}$. Give your answer in the form $y = f(x)$. [7]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

8 (a) Starting from the definition of cosh in terms of exponentials, prove that

$$2 \cosh^2 A = \cosh 2A + 1.$$
 [3]

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The curve C has parametric equations

$$x = 2 \cosh 2t + 3t, \quad y = \frac{3}{2} \cosh 2t - 4t, \quad \text{for } -\frac{1}{2} \leq t \leq \frac{1}{2}.$$

The area of the surface generated when C is rotated through 2π radians about the y -axis is denoted by A .

(b) (i) Show that $A = 10\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} (2 \cosh 2t + 3t) \cosh 2t \, dt.$ [4]

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[7]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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