



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

- [illegible]

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- [illegible]

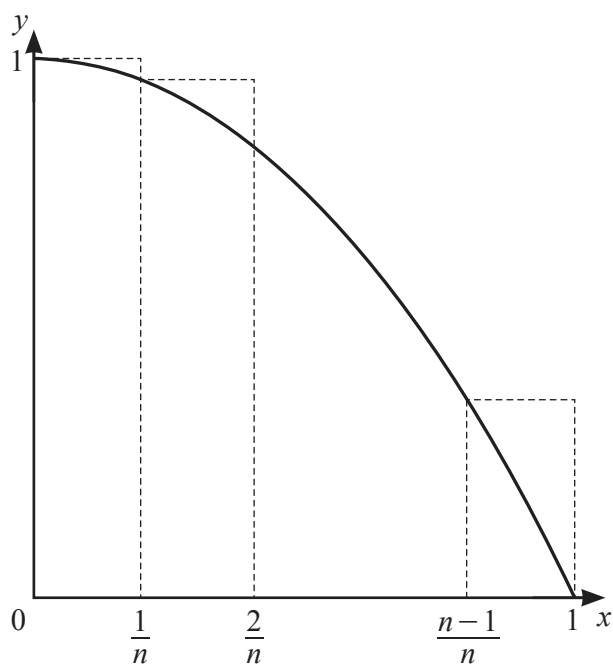
2 Find the solution of the differential equation

$$\frac{dy}{dx} + \frac{4x^3 y}{x^4 + 5} = 6x$$

for which $y = 1$ when $x = 1$. Give your answer in the form $y = f(x)$. [7]

[illegible]

3



The diagram shows the curve with equation $y = 1 - x^2$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of the rectangles, show that

$$\int_0^1 (1-x^2) dx < \frac{4n^2+3n-1}{6n^2}. \quad [4]$$

[illegible]

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(b) Use de Moivre's theorem to show that $\cos 4\theta = 8 \cos^4 \theta - 8 \cos^2 \theta + 1$. [4]

[illegible]

in the form $\cos k\pi$, where k is a rational number. [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

5 The curve C has parametric equations

$$x = 3t + 2t^{-1} + at^3, \quad y = 4t - \frac{3}{2}t^{-1} + bt^3, \quad \text{for } 1 \leq t \leq 2,$$

where a and b are constants.

(a) It is given that $a = \frac{2}{3}$ and $b = -\frac{1}{2}$.

Show that $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \frac{25}{4}(t^2 + t^{-2})^2$ and find the exact length of C . [6]

[illegible]

[4]

[illegible]

$$\mathbf{P} = \begin{pmatrix} 1 & 6 & 6 \\ 0 & 2 & 6 \\ 0 & 0 & -3 \end{pmatrix}.$$

- (a) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [5]

[illegible]

(b) Find the matrix $\mathbf{A}$ such that

$$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{pmatrix}. \quad [4]$$

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

(c) State the eigenvalues and corresponding eigenvectors of $\mathbf{A}^3$. [2]

[illegible]

$$x^2 \frac{d^2 w}{dx^2} + 4x(x+1) \frac{dw}{dx} + (5x^2 + 8x + 2)w = 5x^2 + 4x + 2.$$

(a) Show that

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 5x^2 + 4x + 2. \quad [4]$$

[illegible]

[7]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

8 (a) Starting from the definitions of $\tanh$ and sech in terms of exponentials, prove that

$$1 - \tanh^2 x = \operatorname{sech}^2 x.$$
 [3]

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(b) Using the substitution $u = \tanh x$, or otherwise, find $\int \operatorname{sech}^2 x \tanh^2 x \, dx$. [2]

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It is given that, for $n \geq 0$, $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \tanh^2 x \, dx$.

(c) Show that, for $n \geq 2$,

$$(n + 1)I_n = \left(\frac{4}{5}\right)^3 \left(\frac{3}{5}\right)^{n-2} + (n - 2)I_{n-2}.$$
 [5]

[You may use the result that $\frac{d}{dx}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$.]

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This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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