



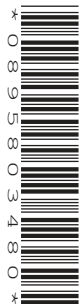
Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

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**FURTHER MATHEMATICS****9231/23**

Paper 2 Further Pure Mathematics 2

October/November 2021**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

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$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I},$$

[6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

$$xy^3 - 4x^3y = 3.$$

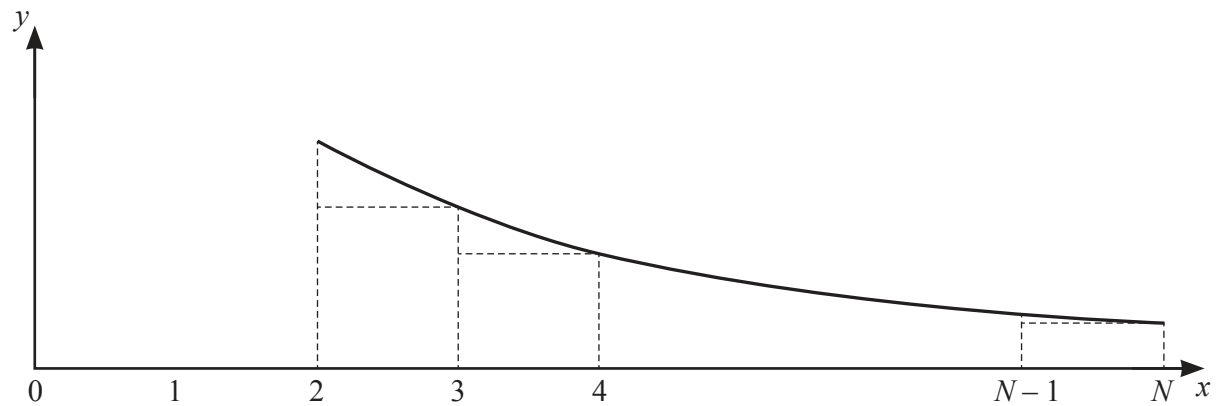
- (a)** Show that, at the point $(-1, 1)$ on C , $\frac{dy}{dx} = 11$. [3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

[5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced and extend across the entire width of the page, providing a guide for letter height and placement. There is no text or other markings on the page.

4



The diagram shows the curve with equation $y = \frac{\ln x}{x^2}$ for $x \geq 2$, together with a set of $(N-2)$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^N \frac{\ln r}{r^2} < \frac{2+3 \ln 2}{4} - \frac{1+\ln N}{N}. \quad [7]$$

[illegible]

(b) Use a similar method to find, in terms of N , a lower bound for $\sum_{r=1}^N \frac{\ln r}{r^2}$. [3]

[illegible]

given that, when $x = 0$, $y = -4$ and $\frac{dy}{dx} = 3$. [11]

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$$x^5 - 10x^4 + 40x^2 - 32 = 0$$

in the form $\operatorname{cosec}(q\pi)$, where q is rational. [4]

[illegible]

7 (a) Show that an appropriate integrating factor for

$$\sqrt{x^2-1} \frac{dy}{dx} + y = x^2 - x\sqrt{x^2-1}$$

is $x + \sqrt{x^2 - 1}$.

[4]

[illegible]

for which $y = 1$ when $x = \frac{5}{4}$. Give your answer in the form $y = f(x)$. [7]

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8 (a) Starting from the definition of cosh in terms of exponentials, prove that

$$2 \cosh^2 A = \cosh 2A + 1.$$
 [3]

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The curve C has parametric equations

$$x = 2 \cosh 2t + 3t, \quad y = \frac{3}{2} \cosh 2t - 4t, \quad \text{for } -\frac{1}{2} \leq t \leq \frac{1}{2}.$$

The area of the surface generated when C is rotated through 2π radians about the y -axis is denoted by A .

(b) (i) Show that $A = 10\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} (2 \cosh 2t + 3t) \cosh 2t \, dt.$ [4]

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This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

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