



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/42

Paper 4 Further Probability & Statistics

October/November 2021

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



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Find a 95% confidence interval for the difference between the mean number of pine trees in regions of this size in countries A and B . [7]

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- 2 It is claimed that the heights of a particular age group of boys follow a normal distribution with mean 125 cm and standard deviation 12 cm. Observations for a randomly chosen group of 60 boys in this age group are summarised in the following table. The table also gives the expected frequencies, correct to 2 decimal places, based on the normal distribution with mean 125 cm and standard deviation 12 cm.

Height, x cm	$x < 100$	$100 \leq x < 110$	$110 \leq x < 120$	$120 \leq x < 130$	$130 \leq x < 140$	$x \geq 140$
Observed frequency	0	3	15	23	11	8
Expected frequency	1.12	5.22	13.97	19.38	13.97	6.34

- (a) Show how the expected frequency for $130 \leq x < 140$ is obtained. [2]

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- (b) Carry out a goodness of fit test, at the 5% significance level, to determine whether the claim is supported by the data. [6]

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[illegible]

3 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} a + \frac{1}{5}x & 0 \leq x < 1, \\ 2a - \frac{1}{5}x & 1 \leq x \leq 2, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(a) Find the value of a . [3]

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(b) Find $E(X^2)$. [2]

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- [illegible]

Test A	46	32	29	12	33	18	25	40
Test B	36	28	49	37	48	35	41	31

- (a) Carry out a Wilcoxon rank-sum test at the 5% significance level to determine whether there is a difference in the population median marks obtained in the two tests. [6]

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This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

5 The random variable X is such that $P(X = r) = kr^2$ for $r = 1, 2, 3, 4$, where k is a constant.

(a) Find the value of k . [1]

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(b) Find the probability generating function $G_X(t)$ of X . [2]

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The random variable Y has probability generating function $G_Y(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$.

The random variable Z is the sum of X and Y .

(c) Assuming that X and Y are independent, find the probability generating function $G_Z(t)$ of Z as a polynomial in t . [3]

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(d) Given that $E(Z) = \frac{13}{3}$, use $G_Z(t)$ to find $\text{Var}(Z)$. [3]

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- 6** A scientist is investigating the masses of a particular type of fish found in lakes A and B . He chooses a random sample of 10 fish of this type from lake A and records their masses, x kg, as follows.

2.1 1.8 0.9 3.0 2.4 2.6 1.8 2.2 1.9 2.5

The scientist also chooses a random sample of 12 fish of this type from lake B , but he only has a summary of their masses, y kg, as follows.

$$\Sigma y = 24.48 \quad \Sigma y^2 = 53.75$$

Test at the 10% significance level whether the mean mass of fish of this type in lake A is greater than the mean mass of fish of this type in lake B . You should state any assumptions that you need to make for the test to be valid. [10]

[illegible]

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