



Cambridge Assessment
International Education

Cambridge International AS & A Level

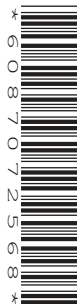
CANDIDATE
NAME

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FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

October/November 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

2 Prove by mathematical induction that, for all positive integers n , $7^{2n} + 97^n - 50$ is divisible by 48. [6]

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1). \quad [5]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- [illegible]

- [illegible]

[illegible]

7

The line l passes through the point P with position vector $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and is parallel to the vector $\mathbf{j} + \mathbf{k}$.

- (b) Find the acute angle between l and Π . [3]

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- (c) Find the position vector of the foot of the perpendicular from P to Π . [4]

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5 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$, where k is a constant.

(a) The matrix **M** represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

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(b) The triangle *ABC* in the *x*–*y* plane is transformed by **M** onto triangle *DEF*.

Find, in terms of k , the single matrix which transforms triangle *DEF* onto triangle *ABC*. [2]

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[illegible]

6 (a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^2 = 36(x^2 - y^2)$$

has polar equation $r^2 = 36 \cos 2\theta$. [3]

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The curve C has polar equation $r^2 = 36 \cos 2\theta$, for $-\frac{1}{4}\pi \leq \theta \leq \frac{1}{4}\pi$.

(b) Sketch C and state the maximum distance of a point on C from the pole. [3]

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[2]

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- [6]

[illegible]

7 The curve C has equation $y = \frac{5x^2}{5x-2}$.

(a) Find the equations of the asymptotes of C . [3]

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(b) Find the coordinates of the stationary points on C . [4]

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13**(c)** Sketch C .**[3]**

(d) Sketch the curve with equation $y = \left| \frac{5x^2}{5x-2} \right|$ and find in exact form the set of values of x for which $\left| \frac{5x^2}{5x-2} \right| < 2$.

[6]

[illegible]

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