



# Cambridge International AS & A Level

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**FURTHER MATHEMATICS****9231/22**

Paper 2 Further Pure Mathematics 2

**October/November 2022****2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

1 (a) Find the set of values of  $k$  for which the system of equations

$$\begin{aligned}x + 2y + 3z &= 1, \\ kx + 4y + 6z &= 0, \\ 7x + 8y + 9z &= 3,\end{aligned}$$

has a unique solution. [3]

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(b) Interpret the situation geometrically in the case where the system of equations does not have a unique solution. [2]

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2 A curve has equation

$$(x + 1)y + y^2 = 2.$$

(a) Show that  $\frac{dy}{dx} = -\frac{2}{3}$  at the point  $(0, -2)$ . [3]

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(b) Find the value of  $\frac{d^2y}{dx^2}$  at the point  $(0, -2)$ . [4]

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**(b)** Using standard results from the list of formulae (MF19), or otherwise, find the Maclaurin’s series for  $e^x + \frac{1}{4}e^{-x}$  up to and including the term in  $x^2$ . [2]

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for which  $x = 3$  when  $t = 1$ . Give your answer in the form  $x = f(t)$ .

[9]

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**5 (a)** Write down the fourth roots of unity. [1]

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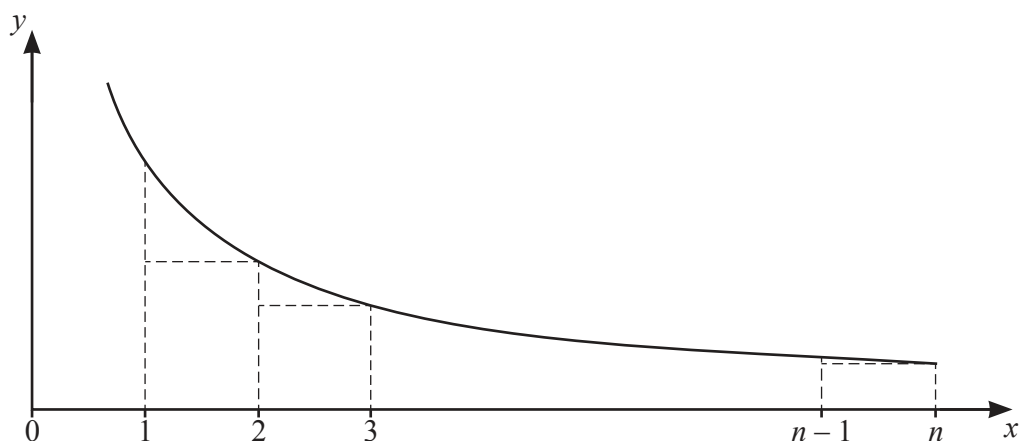
(b) Use de Moivre's theorem to show that

$$\cos 4\theta = 8 \cos^4 \theta - 8 \cos^2 \theta + 1. \quad [4]$$

in the form  $\cos(q\pi)$ , where  $q$  is a rational number. [5]

**[Turn over**

6



The diagram shows the curve  $y = \frac{1}{\sqrt{x^2 + 2x}}$  for  $x > 0$ , together with a set of  $(n-1)$  rectangles of unit width.

By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{r^2+2r}} < \ln(n+1+\sqrt{n^2+2n}) + \frac{1}{3}\sqrt{3} - \ln(2+\sqrt{3}). \quad [10]$$

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, which is otherwise blank.



- 7 (a) It is given that  $\lambda$  is an eigenvalue of the non-singular square matrix  $\mathbf{A}$ , with corresponding eigenvector  $\mathbf{e}$ .

Show that  $\lambda^{-1}$  is an eigenvalue of  $\mathbf{A}^{-1}$  for which  $\mathbf{e}$  is a corresponding eigenvector. [2]

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The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 3 \\ 15 & -4 & 3 \\ 3 & 0 & 2 \end{pmatrix}.$$

- (b) Given that  $-1$  is an eigenvalue of  $\mathbf{A}$ , find a corresponding eigenvector. [2]

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- (c) It is also given that  $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$  are eigenvectors of  $\mathbf{A}$ . Find the corresponding eigenvalues. [2]

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[illegible]

- [illegible]

8 It is given that  $y = \cosh u$ , where  $u > 0$ , and

$$\sqrt{\cosh^2 u - 1} \left( \frac{d^2 u}{dx^2} + \frac{du}{dx} \right) + \cosh u \left( \frac{du}{dx} \right)^2 - 2 \cosh u = 4e^{-x}.$$

(a) Show that

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = 4e^{-x}. \tag{4}$$

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(b) Find  $u$  in terms of  $x$ , given that, when  $x = 0$ ,  $u = \ln 3$  and  $\frac{du}{dx} = 3$ . [10]

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