



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

October/November 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

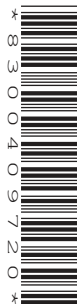
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



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2 (a) Show that the system of equations

$$\begin{aligned}x - y + 2z &= 4, \\ x - y - 3z &= a, \\ x - y + 7z &= 13,\end{aligned}$$

where a is a constant, does not have a unique solution. [2]

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(b) Given that $a = -5$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]

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(c) Given instead that $a \neq -5$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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Find, in terms of e , the length of C .

[6]

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4 (a) Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh^2 x - \sinh^2 x = 1.$$
 [3]

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(b) Show that $\frac{d}{dx}(\tan^{-1}(\sinh x)) = \operatorname{sech} x.$ [3]

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- (c) Sketch the graph of $y = \operatorname{sech} x$, stating the equation of the asymptote. [2]

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- (d) By considering a suitable set of n rectangles of unit width, use your sketch to show that

$$\sum_{r=1}^n \operatorname{sech} r < \tan^{-1}(\sinh n). \quad [3]$$

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- (e) Hence state an upper bound, in terms of π , for $\sum_{r=1}^{\infty} \operatorname{sech} r$. [1]

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$$2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 4x^2 + 3x + 3,$$

given that, when $x = 0$, $y = \frac{dy}{dx} = 0$. [10]

[illegible]

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & -7 \\ 0 & 5 & 7 \\ 0 & 0 & -2 \end{pmatrix}.$$

- (a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

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(b) Use the characteristic equation of $\mathbf{A}$ to show that

$$\mathbf{A}^4 = a\mathbf{A}^2 + b\mathbf{I},$$

where a and b are integers to be determined.

[4]

[illegible]

(b) Show that $(1 + i \tan \theta)^k = \sec^k \theta (\cos k\theta + i \sin k\theta)$, where θ is not an integer multiple of $\frac{1}{2}\pi$. [2]

(c) By considering $\sum_{k=0}^{n-1} (1 + i \tan \theta)^k$, show that

$$\sum_{k=0}^{n-1} \sec^k \theta \sin k\theta = \cot \theta (1 - \sec^n \theta \cos n\theta),$$

provided θ is not an integer multiple of $\frac{1}{2}\pi$. [5]

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(d) Hence find $\sum_{k=0}^{6m-1} 2^k \sin\left(\frac{1}{3}k\pi\right)$ in terms of m . [2]

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$$\int \frac{\theta-1}{\sqrt{1-(\theta-1)^2}} d\theta. \quad [3]$$

(b) Find the solution of the differential equation

$$\theta \frac{dy}{d\theta} - y = \theta^2 \sin^{-1}(\theta - 1),$$

where $0 < \theta < 2$, given that $y = 1$ when $\theta = 1$. Give your answer in the form $y = f(\theta)$. [11]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

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