Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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*8300409720*

**FURTHER MATHEMATICS** **9231/23**

Paper 2 Further Pure Mathematics 2 **October/November 2022**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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**1** Find the Maclaurin's series for $\ln(1+e^x)$ up to and including the term in $x^2$. [5]

**2** **(a)** Show that the system of equations

$$x - y + 2z = 4,$$
$$x - y - 3z = a,$$
$$x - y + 7z = 13,$$

where $a$ is a constant, does not have a unique solution. [2]

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**(b)** Given that $a = -5$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

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**(c)** Given instead that $a \neq -5$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

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**3** The curve $C$ has parametric equations

$$x = \mathrm{e}^t - \tfrac{1}{3}t^3, \quad y = 4\mathrm{e}^{\frac{1}{2}t}(t-2), \qquad \text{for } 0 \leqslant t \leqslant 2.$$

Find, in terms of e, the length of $C$. [6]

**4** **(a)** Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh^2 x - \sinh^2 x = 1.$$ [3]

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**(b)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan^{-1}(\sinh x)\right) = \operatorname{sech} x$. [3]

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**(c)** Sketch the graph of $y = \operatorname{sech} x$, stating the equation of the asymptote. [2]

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**(d)** By considering a suitable set of $n$ rectangles of unit width, use your sketch to show that

$$\sum_{r=1}^{n} \operatorname{sech} r < \tan^{-1}(\sinh n).$$ [3]

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**(e)** Hence state an upper bound, in terms of $\pi$, for $\sum_{r=1}^{\infty} \operatorname{sech} r$. [1]

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**5** Find the particular solution of the differential equation

$$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}+y=4x^2+3x+3,$$

given that, when $x=0,\ y=\frac{\mathrm{d}y}{\mathrm{d}x}=0$. [10]

**6** The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & -7 \\ 0 & 5 & 7 \\ 0 & 0 & -2 \end{pmatrix}.$$

**(a)** Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [7]

**(b)** Use the characteristic equation of **A** to show that

$$\mathbf{A}^4 = a\mathbf{A}^2 + b\mathbf{I},$$

where $a$ and $b$ are integers to be determined. [4]

**7** **(a)** State the sum of the series $1+w+w^2+w^3+\ldots+w^{n-1}$, for $w \neq 1$. [1]

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**(b)** Show that $(1+\mathrm{i}\tan\theta)^k = \sec^k\theta(\cos k\theta+\mathrm{i}\sin k\theta)$, where $\theta$ is not an integer multiple of $\frac{1}{2}\pi$. [2]

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**(c)** By considering $\sum_{k=0}^{n-1}(1+\mathrm{i}\tan\theta)^k$, show that

$$\sum_{k=0}^{n-1}\sec^k\theta\sin k\theta = \cot\theta(1-\sec^n\theta\cos n\theta),$$

provided $\theta$ is not an integer multiple of $\frac{1}{2}\pi$. [5]

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**(d)** Hence find $\sum_{k=0}^{6m-1} 2^k \sin\left(\frac{1}{3}k\pi\right)$ in terms of $m$. [2]

**8** **(a)** Use the substitution $u = 1-(\theta-1)^2$ to find

$$\int \frac{\theta-1}{\sqrt{1-(\theta-1)^2}}\mathrm{d}\theta. \qquad [3]$$

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**(b)** Find the solution of the differential equation

$$\theta\frac{\mathrm{d}y}{\mathrm{d}\theta} - y = \theta^2 \sin^{-1}(\theta-1),$$

where $0 < \theta < 2$, given that $y = 1$ when $\theta = 1$. Give your answer in the form $y = \mathrm{f}(\theta)$. [11]

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## Additional page

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