Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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*2064332093*

**FURTHER MATHEMATICS** **9231/31**

Paper 3 Further Mechanics **October/November 2022**

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ($g$) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (LK/SG) 302027/2

**1** A particle of mass 2 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to a fixed point on a smooth horizontal surface. The particle is moving in a circular path on the surface. The tension in the string is 20 N.

Find how many revolutions the particle makes per minute. [3]

**2** A light elastic string has natural length $a$ and modulus of elasticity $4mg$. One end of the string is fixed to a point $O$ on a smooth horizontal surface. A particle $P$ of mass $m$ is attached to the other end of the string. The particle $P$ is projected along the surface in the direction $OP$. When the length of the string is $\frac{5}{4}a$, the speed of $P$ is $v$. When the length of the string is $\frac{3}{2}a$, the speed of $P$ is $\frac{1}{2}v$.

**(a)** Find an expression for $v$ in terms of $a$ and $g$. [4]

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**(b)** Find, in terms of $g$, the acceleration of $P$ when the stretched length of the string is $\frac{3}{2}a$. [2]

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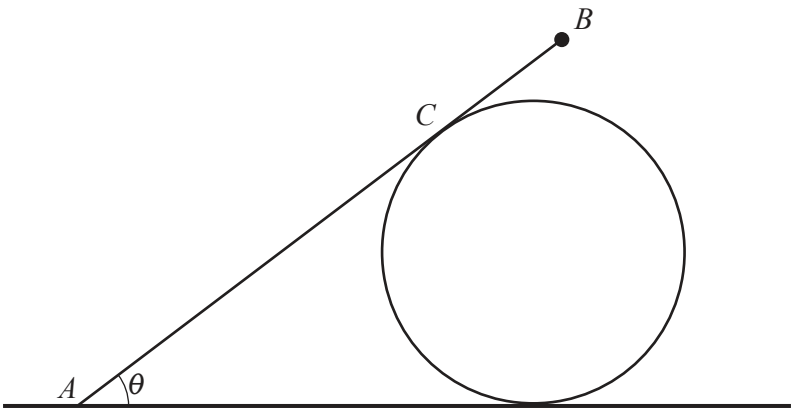
**[Turn over**

**3**




A smooth cylinder is fixed to a rough horizontal surface with its axis of symmetry horizontal. A uniform rod $AB$, of length $4a$ and weight $W$, rests against the surface of the cylinder. The end $A$ of the rod is in contact with the horizontal surface. The vertical plane containing the rod $AB$ is perpendicular to the axis of the cylinder. The point of contact between the rod and the cylinder is $C$, where $AC = 3a$. The angle between the rod and the horizontal surface is $\theta$ where $\tan\theta = \frac{3}{4}$ (see diagram). The coefficient of friction between the rod and the horizontal surface is $\frac{6}{7}$.

A particle of weight $kW$ is attached to the rod at $B$. The rod is about to slip. The normal reaction between the rod and the cylinder is $N$.

**(a)** Show that $N = \frac{8}{15}W(1+2k)$. [2]

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**(b)** Find the value of $k$. [5]

**4** A particle of mass 0.5 kg moves along a horizontal straight line. Its velocity is $v\,\mathrm{m\,s^{-1}}$ at time $t$ s. The forces acting on the particle are a driving force of magnitude 50 N and a resistance of magnitude $2v^2$ N. The initial velocity of the particle is $3\,\mathrm{m\,s^{-1}}$.

**(a)** Find an expression for $v$ in terms of $t$. [7]

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**(b)** Deduce the limiting value of $v$. [1]

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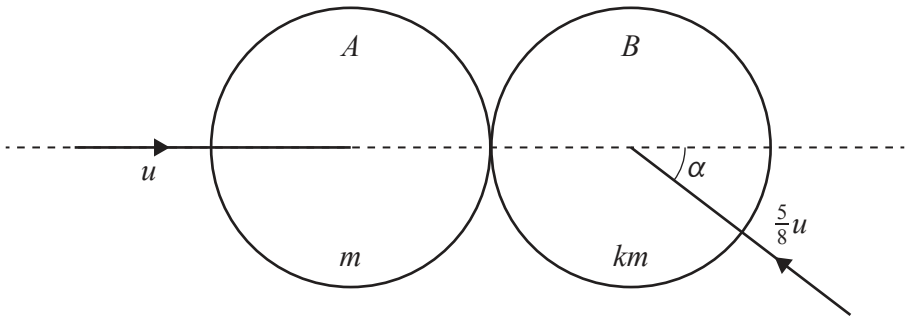
**5** A particle $P$ of mass $m$ is attached to one end of a light inextensible string of length $a$. The other end of the string is attached to a fixed point $O$. The string is held taut with $OP$ horizontal. The particle $P$ is projected vertically downwards with speed $\sqrt{\frac{1}{3}ag}$ and starts to move in a vertical circle. $P$ passes through the lowest point of the circle and reaches the point $Q$ where $OQ$ makes an angle $\theta$ with the downward vertical. At $Q$ the speed of $P$ is $\sqrt{kag}$ and the tension in the string is $\frac{11}{6}mg$.

**(a)** Find the value of $k$ and the value of $\cos\theta$. [4]

At $Q$ the particle $P$ becomes detached from the string.

**(b)** In the subsequent motion, find the greatest height reached by $P$ above the level of the lowest point of the circle. [4]

**6**




Two uniform smooth spheres $A$ and $B$ of equal radii have masses $m$ and $km$ respectively. The two spheres are moving on a horizontal surface with speeds $u$ and $\frac{5}{8}u$ respectively. Immediately before the spheres collide, $A$ is travelling along the line of centres, and $B$'s direction of motion makes an angle $\alpha$ with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$ and $\tan\alpha = \frac{3}{4}$.

After the collision, the direction of motion of $B$ is perpendicular to the line of centres.

**(a)** Find the value of $k$. [4]

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**(b)** Find the loss in the total kinetic energy as a result of the collision. [4]

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**7** A particle $P$ is projected with speed $V\,\text{m}\,\text{s}^{-1}$ at an angle 75° above the horizontal from a point $O$ on a horizontal plane. It then moves freely under gravity.

**(a)** Show that the total time of flight, in seconds, is $\frac{2V}{g}\sin 75°$. [2]

A smooth vertical barrier is now inserted with its lower end on the plane at a distance 15 m from $O$. The particle is projected as before but now strikes the barrier, rebounds and returns to $O$. The coefficient of restitution between the barrier and the particle is $\frac{3}{5}$.

**(b)** Explain why the total time of flight is unchanged. [1]

**(c)** Find an expression for $V$ in terms of $g$. [7]

## Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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