



Cambridge International AS & A Level

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**FURTHER MATHEMATICS****9231/21**

Paper 2 Further Pure Mathematics 2

May/June 2023**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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3

- 1 (a)** Show that the system of equations

$$x + 2y + 3z = 1,$$

$$4x + 5y + 6z = 1,$$

$$7x + 8y + 9z = 1,$$

does not have a unique solution.

[2]

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- (b)** Show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

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$$\frac{dy}{dx} = \frac{1 + 3x + 3y}{3x + 3y - 1}$$

for which $y = 0$ when $x = 1$. Give your answer in the form $a \ln(x+y) + b(x-y) + c = 0$, where a , b and c are constants to be determined. [7]

[illegible]

- 3 (a) By considering the binomial expansion of $(z+z^{-1})^4$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that $\cos^4 \theta = \frac{1}{8}(\cos 4\theta + 4 \cos 2\theta + 3)$. [5]

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- (b) Use the substitution $x = \sin \theta$ to find the exact value of $\int_0^{\frac{1}{2}} (1-x^2)^{\frac{3}{2}} dx$. [3]

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4 The integral I_n is defined by $I_n = \int_0^1 (1+x^5)^n dx$.

(a) By considering $\frac{d}{dx}(x(1+x^5)^n)$, or otherwise, show that

$$(5n+1)I_n = 2^n + 5nI_{n-1}. \quad [5]$$

[illegible]

[4]

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5 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 18 & 5 & -11 \\ 8 & 6 & -4 \\ 32 & 10 & -20 \end{pmatrix}.$$

- (a) Show that the characteristic equation of $\mathbf{A}$ is $\lambda^3 - 4\lambda^2 - 20\lambda + 48 = 0$ and hence find the eigenvalues of $\mathbf{A}$. [4]

[illegible]

[6]

[illegible]

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$. [11]

[illegible]

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- 7 (a) Use the substitution $u = x^2 - 1$ to find $\int \frac{x}{\sqrt{x^2 - 1}} dx$. [3]

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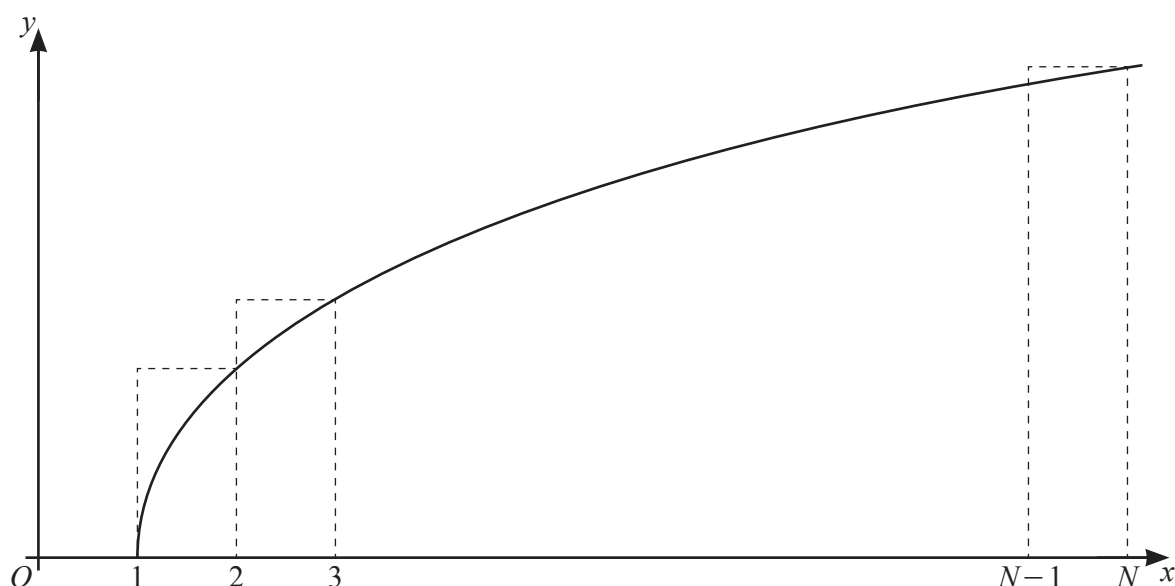
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The diagram shows the curve with equation $y = \cosh^{-1} x$ together with a set of $(N-1)$ rectangles of unit width.

- (b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=2}^N \ln(r + \sqrt{r^2 - 1}) > N \ln(N + \sqrt{N^2 - 1}) - \sqrt{N^2 - 1}. \quad [5]$$

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(c) Use a similar method to find, in terms of N , an upper bound for $\sum_{r=2}^N \ln(r + \sqrt{r^2 - 1})$. [3]

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8 (a) Starting from the definitions of sech and $\tanh$ in terms of exponentials, prove that

$$1 - \operatorname{sech}^2 t = \tanh^2 t.$$
 [3]

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The curve C has parametric equations

$$x = \frac{1}{2} \tanh^2 t + \ln \operatorname{sech} t, \qquad y = 1 + \tanh^4 t, \qquad \text{for } t > 0.$$

(b) Show that $\frac{dy}{dx} = -4 \operatorname{sech}^2 t.$ [5]

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[illegible]

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