



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/42

Paper 4 Further Probability & Statistics

May/June 2023

1 hour 30 minutes

You must answer on the question paper.

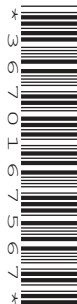
You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

Find a 95% confidence interval for μ . [4]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced, horizontal dotted lines across the entire width of the page. The background is plain white, providing a clear guide for letter height and placement. There are no margins, text, or other markings present.

- 2 The children at two large schools, P and Q , are all given the same puzzle to solve. A random sample of size 10 is taken from the children at school P . Their individual times to complete the puzzle give a sample mean of 9.12 minutes and an unbiased variance estimate of 2.16 minutes². A random sample of size 12 is taken from the children at school Q . Their individual times, x minutes, to complete the puzzle are summarised by

$$\Sigma x = 99.6 \qquad \Sigma (x - \bar{x})^2 = 21.5,$$

where $\bar{x}$ is the sample mean. Times to complete the puzzle are assumed to be normally distributed with the same population variance.

Test at the 5% significance level whether the population mean time taken to complete the puzzle by children at school P is greater than the population mean time taken to complete the puzzle by children at school Q . [8]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Interval	$1 \leq x < 1.5$	$1.5 \leq x < 2$	$2 \leq x < 2.5$	$2.5 \leq x < 3$	$3 \leq x < 3.5$	$3.5 \leq x \leq 4$
Observed frequency	3	3	8	11	13	12

It is required to test the goodness of fit of the distribution with probability density function f given by

$$f(x) = \begin{cases} \frac{1}{24} \left(\frac{4}{x^2} + x^2 \right) & 1 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

The expected frequencies, correct to 4 decimal places, are given in the following table.

Interval	$1 \leq x < 1.5$	$1.5 \leq x < 2$	$2 \leq x < 2.5$	$2.5 \leq x < 3$	$3 \leq x < 3.5$	$3.5 \leq x \leq 4$
Expected frequency	4.4271	a	6.1285	8.4549	b	14.9678

- (a) Show that $a = 4.6007$ and find the value of b . [3]

[illegible]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

A researcher claims that there has been an increase in the median number of employees at technology companies between 2018 and 2022.

- [illegible]

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The researcher notices that the figures for company *G* have been recorded incorrectly. In fact, the number of employees in 2018 was 32 and the number of employees in 2022 was 35.

- (b) Explain, with numerical justification, whether or not the conclusion of the test in part (a) remains the same. [2]

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5 Harry has three coins.

- One coin is biased so that, when it is thrown, the probability of obtaining a head is $\frac{1}{3}$.
- The second coin is biased so that, when it is thrown, the probability of obtaining a head is $\frac{1}{4}$.
- The third coin is biased so that, when it is thrown, the probability of obtaining a head is $\frac{1}{5}$.

The random variable X is the number of heads that Harry obtains when he throws all three coins together.

(a) Find the probability generating function of X . [3]

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Isaac has two fair coins. The random variable Y is the number of heads that Isaac obtains when he throws both of his coins together. The random variable Z is the total number of heads obtained when Harry throws his three coins and Isaac throws his two coins.

(b) Find the probability generating function of Z , expressing your answer as a polynomial in t . [4]

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- This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{3}{28} \left(e^{\frac{1}{2}x} + 4e^{-\frac{1}{2}x} \right) & 0 \leq x \leq 2 \ln 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X . [3]

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The random variable Y is defined by $Y = e^{\frac{1}{2}(X)}$.

(b) Find the probability density function of Y . [3]

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(c) Find the 30th percentile of Y .

[3]

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(d) Find $E(Y^4)$.

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[illegible]

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