



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/43

Paper 4 Further Probability & Statistics

May/June 2023

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

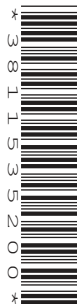
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



1 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{6}\left(x^{-\frac{1}{3}} - x^{-\frac{2}{3}}\right) & 1 \leq x \leq 27, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X . [3]

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The random variable Y is defined by $Y = X^{\frac{1}{3}}$.

(b) Find the probability density function of Y . [3]

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(c) Find the exact value of the median of Y . [2]

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This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

Student	A	B	C	D	E	F	G	H	I
Paper 1	46	73	55	64	86	42	66	68	60
Paper 2	41	66	61	63	90	40	58	42	70

- (a) Carry out a Wilcoxon matched-pairs signed-rank test, at the 5% significance level, to test whether the data supports the teacher's belief. [7]

[illegible]

[illegible]

(a) Test at the 10% significance level whether the data supports the inspector's suspicion. [8]

[illegible]

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(b) Give a reason why it is not necessary to make any assumption about the distributions of the lengths of the rods. [1]

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5 The random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = k(1 + 3t + 4t^2),$$

where k is a constant.

(a) Show that $E(X) = \frac{11}{8}$. [3]

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The random variable Y has probability generating function $G_Y(t)$ given by

$$G_Y(t) = \frac{1}{3}t^2(1 + 2t).$$

The random variables X and Y are independent and $Z = X + Y$.

(b) Find the probability generating function of Z , expressing your answer as a polynomial in t . [2]

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(c) Use your answer to part (b) to find the value of $\text{Var}(Z)$. [3]

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(d) Write down the most probable value of Z . [1]

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	Age of students		
	11–12 years	13–14 years	15–16 years
Grade A	25	16	19
Grade B	28	45	17

- [illegible]

13

The scientist decides instead to use three grades: grade *A* for 16 or more correct, grade *B* for 10 to 15 correct and grade *C* for fewer than 10 correct. The results are shown in the following table.

	Age of students		
	11–12 years	13–14 years	15–16 years
Grade <i>A</i>	25	16	19
Grade <i>B</i>	12	27	11
Grade <i>C</i>	16	18	6

With this second set of data, the test statistic is calculated as 10.91.

(b) Complete the χ^2 -test at the 2.5% significance level for this second set of data. [2]

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(c) State, with a reason, whether you would prefer to use the result from part (a) or part (b) to investigate whether the ability to remember depends on age. [1]

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[illegible]

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