



**Cambridge Assessment**  
International Education

# Cambridge International AS & A Level

CANDIDATE  
NAME

CENTRE  
NUMBER

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**FURTHER MATHEMATICS**

**9231/21**

Paper 2 Further Pure Mathematics 2

**October/November 2023**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

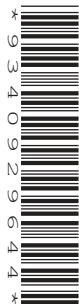
## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.



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$$\begin{array}{rcl} 14x - 4y + 6z & = & 5, \\ x + y + kz & = & 3, \\ -21x + 6y - 9z & = & 14, \end{array}$$

where  $k$  is a constant, does not have a unique solution and interpret this situation geometrically. [4]

[illegible]

[illegible]

[illegible]

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = 27x^2,$$

given that, when  $x = 0$ ,  $y = 2$  and  $\frac{dy}{dx} = -8$ . [10]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



$$x = \frac{2}{3}t^{\frac{3}{2}} - 2t^{\frac{1}{2}}, \quad y = 2t + 5, \quad \text{for } 0 < t \leq 3.$$

- (a)** Find the exact length of  $C$ . [5]

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6 (a) Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\sinh 2x = 2 \sinh x \cosh x.$$
 [3]

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(b) Using the substitution  $u = \sinh x$ , find  $\int \sinh^2 2x \cosh x \, dx$ . [4]

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given that  $y = 4$  when  $x = 0$ . Give your answer in the form  $y = f(x)$ . [7]

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$$\mathbf{A} = \begin{pmatrix} -6 & 2 & 13 \\ 0 & -2 & 5 \\ 0 & 0 & 8 \end{pmatrix}.$$

- [illegible]

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**(b)** Use the characteristic equation of **A** to find  $\mathbf{A}^{-1}$ . [4]

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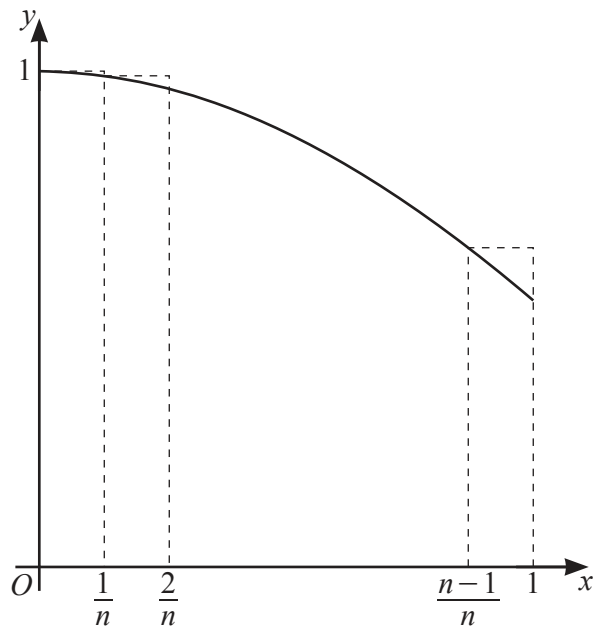
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$$1 + \cos \theta + \cos 2\theta + \dots + \cos(n-1)\theta = \frac{1}{2} \left( 1 - \cos n\theta + \frac{\sin n\theta \sin \theta}{1 - \cos \theta} \right). \quad [7]$$



The diagram shows the curve with equation  $y = \cos x$  for  $0 \leq x \leq 1$ , together with a set of  $n$  rectangles of width  $\frac{1}{n}$ .

(c) By considering the sum of the areas of these rectangles, show that

$$\int_0^1 \cos x \, dx < \frac{1}{2n} \left( 1 - \cos 1 + \frac{\sin 1 \sin \frac{1}{n}}{1 - \cos \frac{1}{n}} \right).$$

[4]

(d) Use a similar method to find, in terms of  $n$ , a lower bound for  $\int_0^1 \cos x \, dx$ . [3]

[illegible]

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