



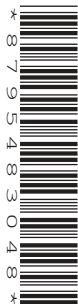
# Cambridge International AS & A Level

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**FURTHER MATHEMATICS****9231/31**

Paper 3 Further Mechanics

**October/November 2023****1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

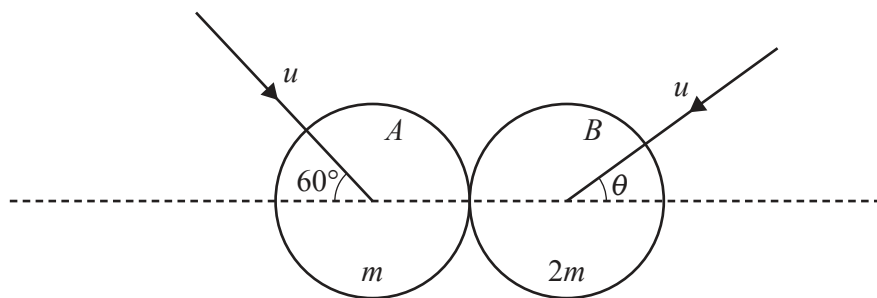
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ( $g$ ) is needed, use  $10 \text{ ms}^{-2}$ .

## INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1



Two uniform smooth spheres  $A$  and  $B$  of equal radii have masses  $m$  and  $2m$  respectively. The two spheres are moving with equal speeds  $u$  on a smooth horizontal surface when they collide. Immediately before the collision,  $A$ 's direction of motion makes an angle of  $60^\circ$  with the line of centres, and  $B$ 's direction of motion makes an angle  $\theta$  with the line of centres (see diagram). The coefficient of restitution between the spheres is  $e$ .

After the collision, the component of the velocity of  $A$  along the line of centres is  $v$  and  $B$  moves perpendicular to the line of centres. Sphere  $A$  now has twice as much kinetic energy as sphere  $B$ .

- (a) Show that  $v = \frac{1}{2}u(4 \cos \theta - 1)$ . [1]

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- (b) Find the value of  $\cos \theta$ . [4]

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(c) Find the value of  $e$ . [2]

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**(b)** Deduce what happens to  $v$  for large values of  $t$ . [1]

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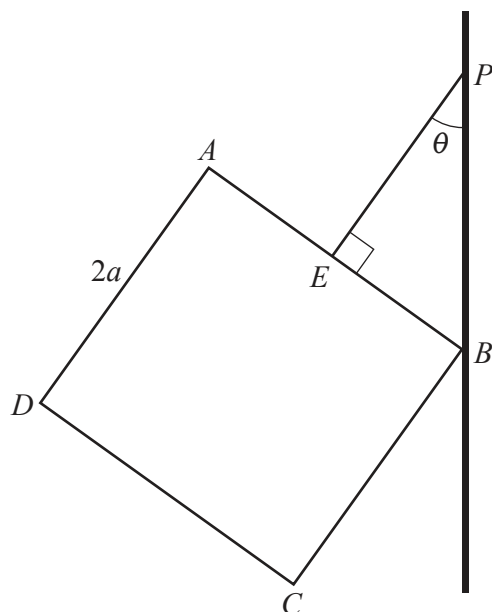
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A uniform square lamina of side  $2a$  and weight  $W$  is suspended from a light inextensible string attached to the midpoint  $E$  of the side  $AB$ . The other end of the string is attached to a fixed point  $P$  on a rough vertical wall. The vertex  $B$  of the lamina is in contact with the wall. The string  $EP$  is perpendicular to the side  $AB$  and makes an angle  $\theta$  with the wall (see diagram). The string and the lamina are in a vertical plane perpendicular to the wall. The coefficient of friction between the wall and the lamina is  $\frac{1}{2}$ .

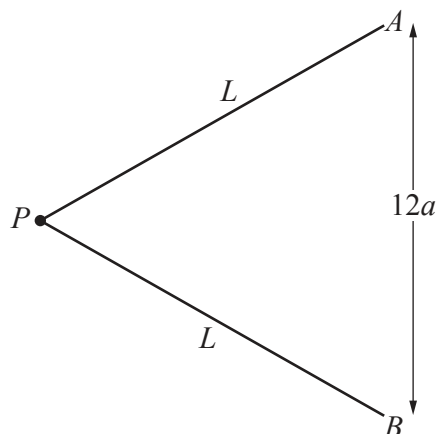
Given that the vertex  $B$  is about to slip up the wall, find the value of  $\tan \theta$ .

[8]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



4



A light elastic string has natural length  $8a$  and modulus of elasticity  $5mg$ . A particle  $P$  of mass  $m$  is attached to the midpoint of the string. The ends of the string are attached to points  $A$  and  $B$  which are a distance  $12a$  apart on a smooth horizontal table. The particle  $P$  is held on the table so that  $AP = BP = L$  (see diagram). The particle  $P$  is released from rest. When  $P$  is at the midpoint of  $AB$  it has speed  $\sqrt{80ag}$ .

- (a)** Find  $L$  in terms of  $a$ .

[5]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

5 A particle  $P$  is projected with speed  $u\text{ ms}^{-1}$  at an angle  $\theta$  above the horizontal from a point  $O$  on a horizontal plane and moves freely under gravity. During its flight  $P$  passes through the point which is a horizontal distance  $3a$  from  $O$  and a vertical distance  $\frac{3}{8}a$  above the horizontal plane. It is given that  $\tan \theta = \frac{1}{3}$ .

(a) Show that  $u^2 = 8ag$ . [2]

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A particle  $Q$  is projected with speed  $V\text{ ms}^{-1}$  at an angle  $\alpha$  above the horizontal from  $O$  at the instant when  $P$  is at its highest point. Particles  $P$  and  $Q$  both land at the same point on the horizontal plane at the same time.

(b) Find  $V$  in terms of  $a$  and  $g$ . [7]

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- 6 A particle  $P$  of mass  $m$  is attached to one end of a light inextensible rod of length  $3a$ . An identical particle  $Q$  is attached to the other end of the rod. The rod is smoothly pivoted at a point  $O$  on the rod, where  $OQ = x$ . The system, of rod and particles, rotates about  $O$  in a vertical plane.

At an instant when the rod is vertical, with  $P$  above  $Q$ , the particle  $P$  is moving horizontally with speed  $u$ . When the rod has turned through an angle of  $60^\circ$  from the vertical, the speed of  $P$  is  $2\sqrt{ag}$ , and the tensions in the two parts of the rod,  $OP$  and  $OQ$ , have equal magnitudes.

- (a) Show that the speed of  $Q$  when the rod has turned through an angle of  $60^\circ$  from the vertical is  $\frac{2x}{3a-x}\sqrt{ag}$ . [2]

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- (b) Find  $x$  in terms of  $a$ . [5]

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(c) Find  $u$  in terms of  $a$  and  $g$ . [4]

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