



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/33

Paper 3 Further Mechanics

October/November 2023

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

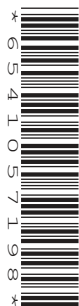
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 ms^{-2} .

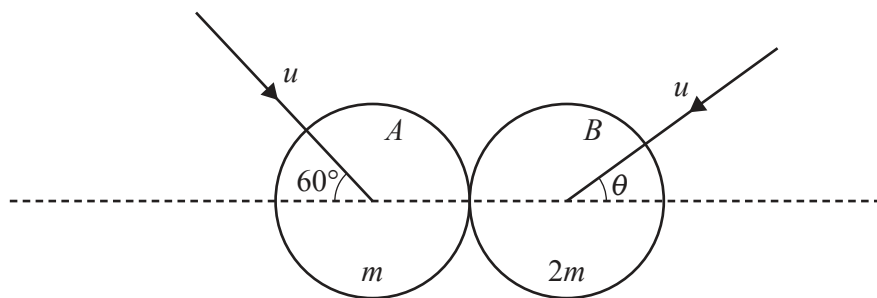
INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



1



Two uniform smooth spheres A and B of equal radii have masses m and $2m$ respectively. The two spheres are moving with equal speeds u on a smooth horizontal surface when they collide. Immediately before the collision, A 's direction of motion makes an angle of 60° with the line of centres, and B 's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is e .

After the collision, the component of the velocity of A along the line of centres is v and B moves perpendicular to the line of centres. Sphere A now has twice as much kinetic energy as sphere B .

- (a) Show that $v = \frac{1}{2}u(4 \cos \theta - 1)$. [1]

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- (b) Find the value of $\cos \theta$. [4]

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(c) Find the value of e . [2]

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(b) Deduce what happens to v for large values of t . [1]

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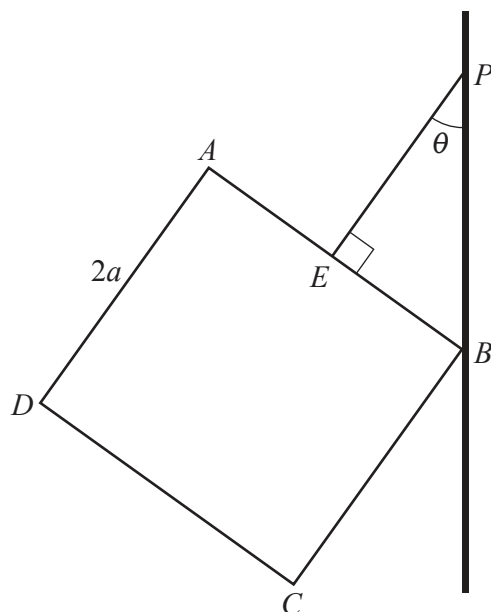
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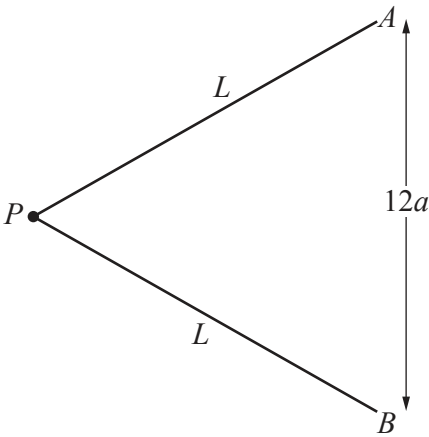


A uniform square lamina of side $2a$ and weight W is suspended from a light inextensible string attached to the midpoint E of the side AB . The other end of the string is attached to a fixed point P on a rough vertical wall. The vertex B of the lamina is in contact with the wall. The string EP is perpendicular to the side AB and makes an angle θ with the wall (see diagram). The string and the lamina are in a vertical plane perpendicular to the wall. The coefficient of friction between the wall and the lamina is $\frac{1}{2}$.

Given that the vertex B is about to slip up the wall, find the value of $\tan \theta$. [8]

[illegible]

4



A light elastic string has natural length $8a$ and modulus of elasticity $5mg$. A particle P of mass m is attached to the midpoint of the string. The ends of the string are attached to points A and B which are a distance $12a$ apart on a smooth horizontal table. The particle P is held on the table so that $AP = BP = L$ (see diagram). The particle P is released from rest. When P is at the midpoint of AB it has speed $\sqrt{80ag}$.

(a) Find L in terms of a . [5]

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5 A particle P is projected with speed $u\text{ ms}^{-1}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. During its flight P passes through the point which is a horizontal distance $3a$ from O and a vertical distance $\frac{3}{8}a$ above the horizontal plane. It is given that $\tan \theta = \frac{1}{3}$.

(a) Show that $u^2 = 8ag$. [2]

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A particle Q is projected with speed $V\text{ ms}^{-1}$ at an angle α above the horizontal from O at the instant when P is at its highest point. Particles P and Q both land at the same point on the horizontal plane at the same time.

(b) Find V in terms of a and g . [7]

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- 6 A particle P of mass m is attached to one end of a light inextensible rod of length $3a$. An identical particle Q is attached to the other end of the rod. The rod is smoothly pivoted at a point O on the rod, where $OQ = x$. The system, of rod and particles, rotates about O in a vertical plane.

At an instant when the rod is vertical, with P above Q , the particle P is moving horizontally with speed u . When the rod has turned through an angle of 60° from the vertical, the speed of P is $2\sqrt{ag}$, and the tensions in the two parts of the rod, OP and OQ , have equal magnitudes.

- (a) Show that the speed of Q when the rod has turned through an angle of 60° from the vertical is $\frac{2x}{3a-x}\sqrt{ag}$. [2]

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- (b) Find x in terms of a . [5]

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(c) Find u in terms of a and g . [4]

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