



Cambridge International AS & A Level

CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER



FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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(d) Given that $\alpha^2 + \beta^2 + \gamma^2 = \frac{1}{3}$, find the value of p . [2]

[Turn over

[illegible]

[illegible]

(b) Express $\frac{3r+4}{r(r+1)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^N \frac{3r+4}{r(r+1)} \left(\frac{1}{4}\right)^{r+1}$$

[4]

[illegible]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)} \left(\frac{1}{4}\right)^{r+1}$. [1]

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4 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 14 & 0 \\ 0 & 1 \end{pmatrix}.$

(a) The matrix **M** represents a sequence of two geometrical transformations in the *x*-*y* plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

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(b) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is **I**. [2]

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- (c) Find the equations of the invariant lines, through the origin, of the transformation represented by **M**. [5]

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- (d) The triangle *ABC* in the *x-y* plane is transformed by **M** onto triangle *DEF*.
Given that the area of triangle *DEF* is 28 cm^2 , find the area of triangle *ABC*. [2]

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respectively, relative to the origin O .

- (a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

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[illegible][illegible]

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- (c) Sketch C , stating the coordinates of the point of intersection with the y -axis and labelling the asymptotes. [3]

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- (d) (i) Sketch the curve with equation $y = \left| \frac{x^2 + ax + 1}{x + 2} \right|$. [2]

- (ii) On your sketch in part (i), draw the line $y = a$. [1]

- (iii) It is given that $\left| \frac{x^2 + ax + 1}{x + 2} \right| < a$ for $-5 - \sqrt{14} < x < -3$ and $-5 + \sqrt{14} < x < 3$.

Find the value of a . [2]

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- (b) Using the substitution $u = \pi - \theta$, or otherwise, find the area of the region enclosed by C and the initial line. [7]

(c) Show that, at the point of C furthest from the initial line,

$$2(\pi - \theta) \tan^{-1}(\pi - \theta) \cot \theta - \frac{\pi - \theta}{1 + (\pi - \theta)^2} - \tan^{-1}(\pi - \theta) = 0$$

and verify that this equation has a root for θ between 1.2 and 1.3. [5]

[illegible]

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