



Cambridge Assessment
International Education



Cambridge International AS & A Level

CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



BLANK PAGE

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN





1 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} k & 1 & 0 \\ 6 & 5 & 2 \\ -1 & 3 & -k \end{pmatrix},$$

where k is a real constant.

(a) Show that **A** is non-singular. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Given that $\mathbf{A}^{-1} = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 0 & 0 \\ -\frac{23}{2} & \frac{1}{2} & 3 \end{pmatrix}$, find the value of k . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





(a) Find a cubic equation whose roots are $\alpha^2 + 1, \beta^2 + 1, \gamma^2 + 1$. [3]

[illegible]



(b) Find the value of $(\alpha^2 + 1)^2 + (\beta^2 + 1)^2 + (\gamma^2 + 1)^2$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the value of $(\alpha^2 + 1)^3 + (\beta^2 + 1)^3 + (\gamma^2 + 1)^3$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





3 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 7 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x - y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the equations of the invariant lines, through the origin, of the transformation represented by $\mathbf{M}$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





DO NOT WRITE IN THIS MARGIN

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The triangle DEF in the x - y plane is transformed by $\mathbf{M}$ onto triangle PQR .

- (c) Given that the area of triangle PQR is 35 cm^2 , find the area of triangle DEF . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





8

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1). \quad [5]$$
[illegible]





- 5 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} - 2\mathbf{k})$ and $\mathbf{r} = -3\mathbf{i} + 4\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.

- (a) Find the shortest distance between l_1 and l_2 . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The plane Π_1 contains l_1 and is parallel to l_2 .

- (b) Obtain an equation of Π_1 in the form $px + qy + rz = s$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





It is given that the line of intersection of the planes Π_1 and Π_2 passes through the point $(0, 0, 2)$ and is parallel to the vector $\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$.

Obtain an equation of Π_γ in the form $ax+by+cz=d$. [3]

[illegible]

- (a) Show that C has no vertical asymptotes and state the equation of the horizontal asymptote. [2]

[illegible]

- (b)** Find the coordinates of any stationary points on C . [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, which is otherwise blank.



13



(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

.....

(d) Sketch $y^2 = \frac{x+1}{x^2+3}$, stating the coordinates of the stationary points and the intersections with the axes. [4]





7 The curve C has polar equation $r^2 = \sin 2\theta \cos \theta$, for $0 \leq \theta \leq \pi$.

(a) Sketch C and state the equation of the line of symmetry.

[3]

(b) Find a Cartesian equation for C .

[3]





[4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.



[6]

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]



BLANK PAGE

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN





BLANK PAGE

DO NOT WRITE IN THIS MARGIN





BLANK PAGE

DO NOT WRITE IN THIS MARGIN

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

