



Cambridge International AS & A Level

CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER



FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

[illegible]

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(a) Show that $\frac{dy}{dx} = -t + \sqrt{1-t^2} \cos^{-1} t$. [3]

[illegible]

[4]

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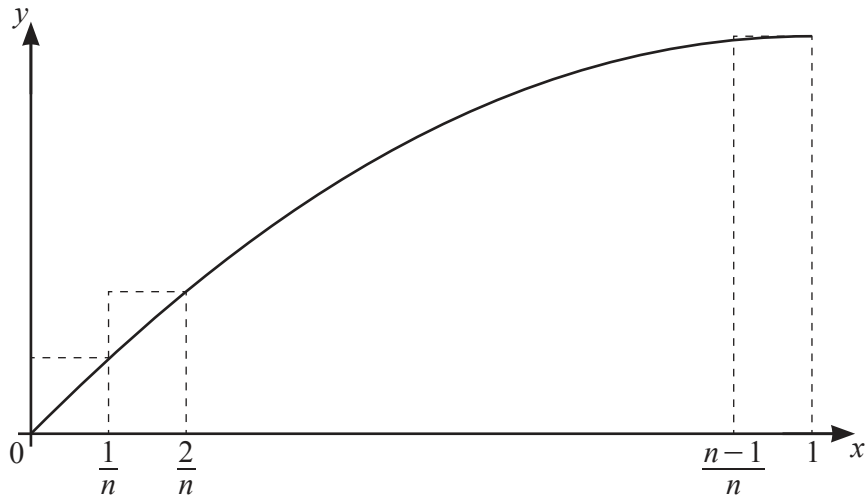
$$(n-1)I_n = \left(\frac{3}{5}\right)^{n-2} \left(\frac{4}{5}\right) + (n-2)I_{n-2}. \quad [5]$$

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[3]

[illegible]

5



The diagram shows the curve with equation $y = 2x - x^2$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (a) By considering the sum of the areas of these rectangles, show that $\int_0^1 (2x - x^2) dx < U_n$, where

$$U_n = \left(1 + \frac{1}{n}\right) \left(\frac{2}{3} - \frac{1}{6n}\right). \quad [5]$$

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(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 (2x - x^2) dx$. [4]

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(c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

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6 (a) Show that $(\cosh x + \sinh x)^{\frac{1}{2}} = e^{\frac{1}{2}x}$. [2]

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(b) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5(\cosh x + \sinh x)^{\frac{1}{2}},$$

given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$. [10]

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[illegible]

$$x \frac{dy}{dx} - y = x^2 \sinh^{-1} x,$$

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8 (a) Find the set of values of a for which the system of equations

$$\begin{aligned} 6x + ay &= 3, \\ 2x - y &= 1, \\ x + 5y + 4z &= 2 \end{aligned}$$

has a unique solution. [2]

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(b) Show that the system of equations in part (a) is consistent for all values of a . [3]

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$$\mathbf{A} = \begin{pmatrix} 6 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 5 & 4 \end{pmatrix}.$$

- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $(14\mathbf{A} + 24\mathbf{I})^2 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

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(d) Use the characteristic equation of $\mathbf{A}$ to show that

$$(14\mathbf{A} + 24\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A} + b\mathbf{I})^2,$$

where b is an integer to be determined. [4]

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