



Cambridge Assessment
International Education



Cambridge International AS & A Level

CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



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3



1 Find the exact value of $\int_2^{\frac{7}{2}} \frac{1}{\sqrt{4x-x^2-1}} dx$.

[5]




$$x = \cosh t, \quad y = \sinh t, \quad \text{for } 0 < t \leq \frac{3}{5}.$$

(a) Show that $s = \int_0^{\frac{3}{5}} \sqrt{\cosh 2t} \, dt$.

[4]

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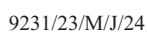
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[5]

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(a) Show that, at the point $(-2, -1)$ on C , $\frac{dy}{dx} = -\frac{1}{2}$. [3]

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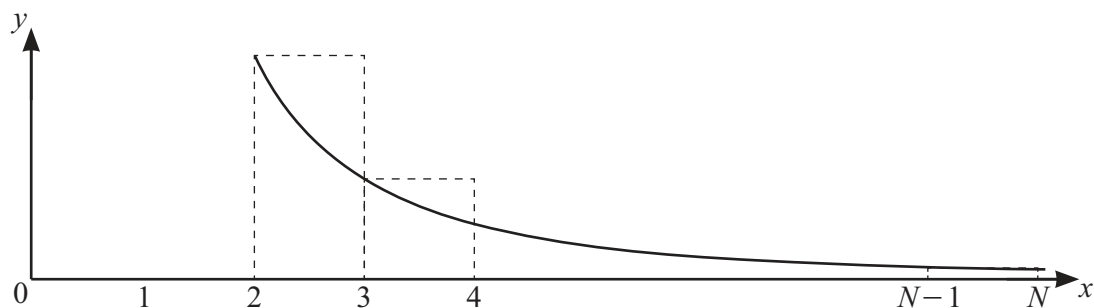


[5]

[Turn over



4



The diagram shows the curve with equation $y = x^{-2}$ for $2 \leq x \leq N$ together with a set of $(N-2)$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^N \frac{1}{r^2} > \frac{3}{2} - \frac{1}{N} + \frac{1}{N^2}. \quad [5]$$

[illegible]



(b) Use a similar method to find, in terms of N , an upper bound for $\sum_{r=1}^N \frac{1}{r^2}$. [3]

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(c) Deduce lower and upper bounds for $\sum_{r=1}^{\infty} \frac{1}{r^2}$. [2]

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$$\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 25x = 338 \sin t. \quad [7]$$
[illegible]


$$x \approx R \sin(t - \phi),$$

[3]

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6 (a) Show that $\sum_{r=1}^n z^{4r} = \frac{z^{4n+2} - z^2}{z^2 - z^{-2}}$, for $z^2 \neq z^{-2}$. [2]

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(b) By letting $z = \cos \theta + i \sin \theta$, show that, if $\sin 2\theta \neq 0$,

$$\sum_{r=1}^n \sin(4r\theta) = \frac{\cos 2\theta - \cos(4n+2)\theta}{2 \sin 2\theta}. \quad [5]$$

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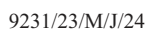
[Turn over


$$\frac{d}{dx} \left(\frac{x}{2} \sqrt{x^2 - 9} - \frac{9}{2} \cosh^{-1} \frac{x}{3} \right) = \sqrt{x^2 - 9}. \quad [3]$$
$$x \frac{dy}{dx} - y = x^2 \sqrt{x^2 - 9},$$

given that $y = 1$ when $x = 3$. Give your answer in the form $y = f(x)$. [9]



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[Turn over



- (a) Find Cartesian equations of Π_1 and Π_2 . [3]

[illegible]

- (b) Express the vector equation of l in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{a} + \lambda \mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are vectors to be determined, and hence show that for points on l , $\frac{1}{2}x + \frac{1}{12}y = 1$ and $z = 0$. [2]

[illegible]


$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ \frac{1}{2} & \frac{1}{12} & 0 \end{pmatrix}.$$

- [illegible]



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