



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



1 The matrix **M** represents the sequence of two transformations in the x - y plane given by a stretch parallel to the x -axis, scale factor k ($k \neq 0$), followed by a shear, x -axis fixed, with $(0, 1)$ mapped to $(k, 1)$.

(a) Show that $\mathbf{M} = \begin{pmatrix} k & k \\ 0 & 1 \end{pmatrix}$. [4]

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(b) The transformation represented by **M** has a line of invariant points.
Find, in terms of k , the equation of this line. [3]

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- (c) Find, in terms of k , a matrix which transforms P onto S . [1]

[illegible]

- (d) Given that the area of P is $3k^2$ units², find the possible values of k . [2]

This image shows a full page of white paper designed for handwriting practice. It features 20 evenly spaced, horizontal dotted lines that run across the entire width of the page. The lines are thin and light gray, providing a guide for letter height and placement without being distracting. There is no text or other markings on the page.


$$\frac{d^n}{dx^n}(\tan^{-1}x) = P_n(x)(1+x^2)^{-n},$$

[6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]



- [5]

[illegible]



(b) Find the value of $\alpha^5 + \beta^5 + \gamma^5 + \delta^5$. [3]

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(c) Find the value of $\alpha^8 + \beta^8 + \gamma^8 + \delta^8$. [2]

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- 4 (a) Use the method of differences to find $\sum_{r=1}^n \frac{5k}{(5r+k)(5r+5+k)}$ in terms of n and k , where k is a positive constant. [4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.



It is given that $\sum_{r=1}^{\infty} \frac{5k}{(5r+k)(5r+5+k)} = \frac{1}{3}$.

(b) Find the value of k . [2]

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(c) Hence find $\sum_{r=n}^{n^2} \frac{5k}{(5r+k)(5r+5+k)}$ in terms of n . [2]

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5 (a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^2 = 6xy$$

has polar equation $r^2 = 3 \sin 2\theta$. [2]

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The curve C has polar equation $r^2 = 3 \sin 2\theta$, for $0 \leq \theta \leq \frac{1}{2}\pi$.

(b) Sketch C and state the maximum distance of a point on C from the pole. [3]

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(c) Find the area of the region enclosed by C .

[2]

(d) Find the maximum distance of a point on C from the initial line.

[6]



[2]

[4]

[illegible]



(c) Sketch C , stating the coordinates of any intersections with the axes.

[5]

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(d) Sketch the curve with equation $y = \left| \frac{4x^2 + x + 1}{2x^2 - 7x + 3} \right|$ and state the set of values of k for which

$$\left| \frac{4x^2 + x + 1}{2x^2 - 7x + 3} \right| = k \text{ has 4 distinct real solutions.}$$

[2]





- 7 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 9\mathbf{k} + \mu(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ respectively. The plane Π_1 contains l_1 and is parallel to l_2 .

(a) Find the equation of Π_1 , giving your answer in the form $ax + by + cz = d$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

The plane Π_2 contains l_2 and the point with coordinates $(2, -1, 7)$.

(b) Find the acute angle between Π_1 and Π_2 . [4]

[illegible]



The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2 .

(c) Find a vector equation for PQ .

[7]

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly apart, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text on the page.

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