

Cambridge Assessment  
International Education

# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.



- (a)** Show that  $\mathbf{M} = \begin{pmatrix} k & k \\ 0 & 1 \end{pmatrix}$ . [4]

[illegible]

- Find, in terms of  $k$ , the equation of this line. [3]

[illegible]



(c) Find, in terms of  $k$ , a matrix which transforms  $P$  onto  $S$ . [1]

[illegible]

(d) Given that the area of  $P$  is  $3k^2$  units<sup>2</sup>, find the possible values of  $k$ . [2]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.


$$\frac{d^n}{dx^n}(\tan^{-1}x) = P_n(x)(1+x^2)^{-n},$$

[6]

[illegible]



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Handwriting practice area with horizontal dotted lines.





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(b) Find the value of  $\alpha^5 + \beta^5 + \gamma^5 + \delta^5$ . [3]

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(c) Find the value of  $\alpha^8 + \beta^8 + \gamma^8 + \delta^8$ . [2]

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[4]

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It is given that  $\sum_{r=1}^{\infty} \frac{5k}{(5r+k)(5r+5+k)} = \frac{1}{3}$ .

(b) Find the value of  $k$ .

[2]

This image shows a blank sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

(c) Hence find  $\sum_{r=n}^{n^2} \frac{5k}{(5r+k)(5r+5+k)}$  in terms of  $n$ .

[2]

[illegible]



5 (a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^2 = 6xy$$

has polar equation  $r^2 = 3 \sin 2\theta$ .

[2]

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The curve  $C$  has polar equation  $r^2 = 3 \sin 2\theta$ , for  $0 \leq \theta \leq \frac{1}{2}\pi$ .

(b) Sketch  $C$  and state the maximum distance of a point on  $C$  from the pole.

[3]





(c) Find the area of the region enclosed by  $C$ .

[2]

**(d)** Find the maximum distance of a point on  $C$  from the initial line.

[6]





[2]

[illegible]

[4]

This image shows a full page of handwriting practice paper. It features 18 horizontal rows, each defined by two parallel dashed lines. The rows are evenly spaced and extend across the entire width of the page, providing a guide for letter height and placement. There is no text or other markings on the page.



(c) Sketch  $C$ , stating the coordinates of any intersections with the axes.

[5]

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(d) Sketch the curve with equation  $y = \left| \frac{4x^2 + x + 1}{2x^2 - 7x + 3} \right|$  and state the set of values of  $k$  for which

$$\left| \frac{4x^2 + x + 1}{2x^2 - 7x + 3} \right| = k \text{ has 4 distinct real solutions.}$$

[2]





- 7 The lines  $l_1$  and  $l_2$  have equations  $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k})$  and  $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 9\mathbf{k} + \mu(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$  respectively. The plane  $\Pi_1$  contains  $l_1$  and is parallel to  $l_2$ .

**(a)** Find the equation of  $\Pi_1$ , giving your answer in the form  $ax + by + cz = d$ . [4]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

The plane  $\Pi_2$  contains  $l_2$  and the point with coordinates  $(2, -1, 7)$ .

(b) Find the acute angle between  $\Pi_1$  and  $\Pi_2$ . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.



The point  $P$  on  $l_1$  and the point  $Q$  on  $l_2$  are such that  $PQ$  is perpendicular to both  $l_1$  and  $l_2$ .

(c) Find a vector equation for  $PQ$ .

[7]

[illegible]



## Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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