

Cambridge Assessment
International Education

Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



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$$\begin{aligned}x + 5y + 6z &= 1, \\ kx + 2y + 2z &= 2, \\ -3x + 4y + 8z &= 3,\end{aligned}$$

has a unique solution and interpret this situation geometrically. [4]

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$$x = 1 + \frac{1}{t} \quad \text{and} \quad y = \cos^{-1} t \quad \text{for } 0 < t < 1.$$

(a) Show that $\frac{dy}{dx} = \frac{t^2}{\sqrt{1-t^2}}$. [2]

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[illegible]



- 3 A curve has equation $y = e^x$ for $\ln \frac{4}{3} \leq x \leq \ln \frac{12}{5}$. The area of the surface generated when the curve is rotated through 2π radians about the x -axis is denoted by A .

- (a) Use the substitution $u = e^x$ to show that

$$A = 2\pi \int_{\frac{4}{3}}^{\frac{12}{5}} \sqrt{1+u^2} \, du. \quad [2]$$

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- (b) Use the substitution $u = \sinh v$ to show that

$$A = \pi \left(\frac{904}{225} + \ln \frac{5}{3} \right). \quad [6]$$

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4 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -11 & 1 & 8 \\ 0 & -2 & 0 \\ -16 & 1 & 13 \end{pmatrix}.$$

- (a) Show that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$ and state the corresponding eigenvalue. [2]

[illegible]

- (b)** Show that the characteristic equation of $\mathbf{A}$ is $\lambda^3 - 19\lambda - 30 = 0$ and hence find the other eigenvalues of $\mathbf{A}$. [3]

[illegible]



(c) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$.

[4]

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5 Find the particular solution of the differential equation

$$6\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + x = t^2 + t + 1,$$

given that, when $t = 0$, $x = 12$ and $\frac{dx}{dt} = -6$. [10]

[illegible]



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(b) Use a similar method to find, in terms of N , an upper bound U_N for $\int_0^1 \left(\frac{1}{2}\right)^x \mathrm{d}x$. [4]

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(c) Find the least value of N such that $U_N - L_N \leq 10^{-3}$. [2]

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- (d) Given that $\int_0^1 \left(\frac{1}{2}\right)^x dx = \frac{1}{2 \ln 2}$, use the value of N found in part (c) to find upper and lower bounds for $\ln 2$. [4]

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- 7 (a) Show that an appropriate integrating factor for

$$\sqrt{x^2 + 16} \frac{dy}{dx} + y = x\sqrt{x^2 + 16}$$

$$\text{is } \frac{1}{4}x + \frac{1}{4}\sqrt{x^2 + 16}. \quad [4]$$
This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.


$$\sqrt{x^2 + 16} \frac{dy}{dx} + y = x\sqrt{x^2 + 16}$$

[6]

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Let $I_n = \int_0^{\frac{1}{4}\pi} \cos^n \theta \, d\theta$.

(b) Show that

$$nI_n = 2^{-\frac{1}{2}n} + (n-1)I_{n-2}. \quad [4]$$

[illegible]



(c) Using the results given in parts (a) and (b), find the exact value of I_9 .

[5]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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