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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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FURTHER MATHEMATICS **9231/13**

Paper 1 Further Pure Mathematics 1 **October/November 2025**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n}\left(8r^3+12r^2+4r+3\right)$ in terms of n, simplifying your answer. [3]

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(b) Show that

$$\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$$

and hence use the method of differences to find $\sum_{r=1}^{n} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [1]

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2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix}.$$

(a) Give full details of the geometrical transformation in the x-y plane represented by **A**. [2]

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(b) Give full details of the geometrical transformation in the x-y plane represented by **B**. [2]

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The triangle DEF in the x-y plane is transformed by **AB** onto triangle PQR.

(c) Show that the triangles DEF and PQR have the same area. [2]

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(d) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **AB**. [5]

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3 Prove by mathematical induction that, for every positive integer n,

$$\frac{\mathrm{d}^{2n-1}}{\mathrm{d}x^{2n-1}}(x\cos x) = (-1)^n(x\sin x - (2n-1)\cos x).$$ [7]

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4 The quartic equation $x^4 + x^3 + x^2 + x + 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Show that a quartic equation with roots $2\alpha + 1, 2\beta + 1, 2\gamma + 1, 2\delta + 1$ is

$$y^4 - 2y^3 + 4y^2 + 2y + 11 = 0.$$ [4]

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The sum $(2\alpha+1)^n + (2\beta+1)^n + (2\gamma+1)^n + (2\delta+1)^n$ is denoted by S_n.

(b) Find the value of S_2. [2]

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(c) Given that $S_3 = -22$, find the value of S_4. [2]

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5 The plane Π_1 has equation $\mathbf{r} = -3\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$. [4]

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(b) Find the perpendicular distance from the point with position vector $-\mathbf{i} - 2\mathbf{k}$ to Π_1. [3]

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(c) The plane Π_2 has equation $3x+2y-z=14$.

Find a vector equation of the line of intersection of Π_1 and Π_2. [4]

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6 The curve C has polar equation $r = \sin 3\theta$, for $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.

(a) Sketch C and state the equation of the line of symmetry. [3]

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(b) Find the exact value of the area of the region enclosed by C. [4]

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In parts **(c)** and **(d)** you may use the identity $\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$.

(c) Find the maximum distance of a point on C from the initial line. [5]

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(d) Find a Cartesian equation for C. [3]

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7 The curve C has equation $y = \dfrac{x+2}{x^2+3x+1}$.

(a) Find the equations of the asymptotes of C. [2]

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(b) Show that C has no stationary points. [3]

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(c) Sketch C, stating the coordinates of the intersections with the axes. [3]

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(d) Sketch the curve with equation $y = \left|\dfrac{x+2}{x^2+3x+1}\right|$. [2]

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(e) Find in exact form the set of values of x for which $\left|\frac{x+2}{x^2+3x+1}\right| > 2$. [4]

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Additional page

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