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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3146099302

FURTHER MATHEMATICS **9231/14**

Paper 1 Further Pure Mathematics 1 **October/November 2025**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Prove by mathematical induction that $\left(\frac{6}{5}\right)^n \geqslant 1 + \frac{1}{5}n$ for all positive integers n. [5]

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2 The cubic equation $x^3 + bx^2 + cx + d = 0$, where $d \neq 0$, has roots α, β, γ such that $\gamma = \frac{1}{\beta}$.

(a) Show that $\beta + \frac{1}{\beta} = d - b$. [3]

(b) Show also that $\beta + \frac{1}{\beta} = \frac{1-c}{d}$. [2]

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(c) It is given that $b = 3$, $c = -3$ and $d > 0$.

(i) Find the value of d. [2]

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(ii) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

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3 **(a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n}(r+1)(r+2)(r+3) = \tfrac{1}{4}n\left(n^3 + bn^2 + cn + d\right),$$

where b, c and d are integers to be determined. [3]

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(b) Express $\frac{2}{(r+1)(r+2)(r+3)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+2)(r+3)}.$$ [5]

(c) State the value of $\sum_{r=1}^{\infty} \frac{2}{(r+1)(r+2)(r+3)}$. [1]

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4 The points A, B, C have position vectors

$$3\mathbf{i}+5\mathbf{j}+5\mathbf{k}, \qquad 2\mathbf{i}+2\mathbf{j}+2\mathbf{k}, \qquad 2\mathbf{i}+\mathbf{j}-\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

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(b) The point D has position vector $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

Find the shortest distance between the lines AB and CD. [5]

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5 **(a)** **(i)** By writing $\cos\left(\frac{1}{12}\pi\right)$ as $\cos\left(\frac{1}{4}\pi - \frac{1}{6}\pi\right)$, show that $\cos\left(\frac{1}{12}\pi\right) = \frac{1}{4}\left(\sqrt{6} + \sqrt{2}\right)$. [1]

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(ii) Show also that $\sin\left(\frac{1}{12}\pi\right) = \frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right)$. [1]

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The matrix **M** is such that $\mathbf{M} = \frac{1}{4}\begin{pmatrix}\sqrt{6}+\sqrt{2} & \sqrt{2}-\sqrt{6}\\ \sqrt{6}-\sqrt{2} & \sqrt{6}+\sqrt{2}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$.

(b) The matrix **M** represents a sequence of two geometrical transformations in the x-y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

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(c) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

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(d) Given that $y = mx$ is an invariant line of the transformation represented by $\mathbf{M}$, show that

$$m^2 \sin\left(\tfrac{1}{12}\pi\right) + 2m\cos\left(\tfrac{1}{12}\pi\right) - \sin\left(\tfrac{1}{12}\pi\right) = 0$$

and find the values of m in the form $a\cot\left(\tfrac{1}{12}\pi\right) + b\,\mathrm{cosec}\left(\tfrac{1}{12}\pi\right)$, where a and b are integers to be determined. [6]

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6 The curve C has polar equation $r = \cos\frac{1}{2}\theta$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C. [2]

In parts **(b)** and **(c)** you may use the identities $\sin\theta \equiv 2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta$ and $\cos\theta \equiv 2\cos^2\frac{1}{2}\theta - 1$.

(b) Find the exact value of the area of the region enclosed by C and the initial line. [4]

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(c) Find the maximum distance of a point on C from the initial line. Give your answer in the form $p\sqrt{q}$, where p and q are rational numbers to be determined. [6]

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7 The curve C has equation $y = \dfrac{10x^2 - 11x - 18}{10x - 18}$.

(a) Find the equations of the asymptotes of C. [3]

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(b) Show that C has no stationary points. [4]

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(c) Sketch C, stating the coordinates of the points of intersection with the axes. [3]

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(d) Sketch the curve with equation $y = \left| \frac{10x^2 - 11x - 18}{10x - 18} \right|$. [2]

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(e) Given that $\left|\frac{10x^2-11x-18}{10x-18}\right| < 4$ for $\frac{-29-\sqrt{4441}}{20} < x < p$ and $\frac{-29+\sqrt{4441}}{20} < x < q$, find the values of p and q. [4]

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