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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **October/November 2025**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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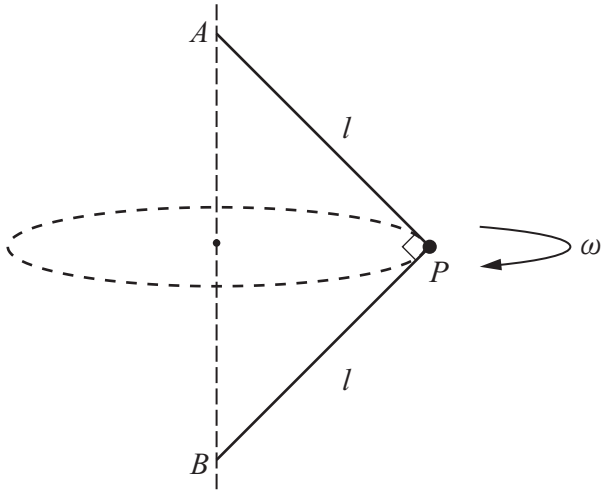
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1 A particle P of mass m is attached to two light inextensible strings each of length l. The end of one string is attached to a fixed point A and the end of the other string is attached to a fixed point B, with A vertically above B. Angle APB is a right angle. The particle P rotates in a horizontal circle at a constant angular speed ω with both strings taut (see diagram).



Find the tension in string AP in terms of m, g, l and ω. [4]

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2 One end of a light elastic string of natural length a and modulus of elasticity $2mg$ is attached to a fixed point A on a rough horizontal surface. The other end of the string is attached to a particle P of mass m. The particle and string rest on the surface. The coefficient of friction between P and the surface is μ. The particle P is initially held in equilibrium at a distance $\frac{4}{3}a$ from A. The particle is then released from rest.

(a) Given that the string never becomes slack, find the minimum value of μ. [3]

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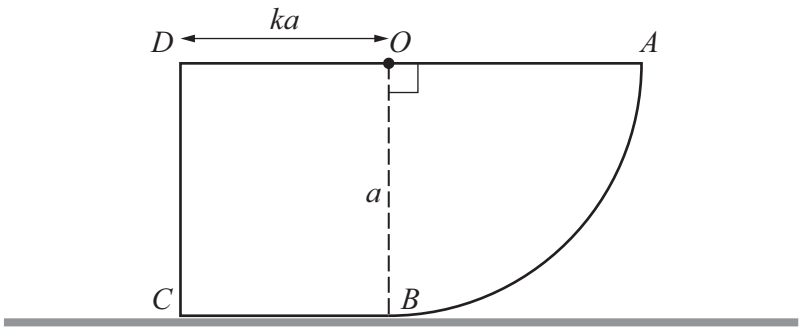
It is now given that $\mu = \frac{1}{2}$.

(b) Find the extension of the string when the particle comes to rest. [3]

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3 A uniform lamina *OABCD* is in the form of a rectangle, *OBCD*, joined along the edge *OB* to a quarter circle *OAB*. The length of *DO* is *ka* and the length of *OB* is *a*. The lamina rests in a vertical plane with its edge *CB* on a horizontal surface (see diagram).



(a) Find, in terms of k, a and π, an expression for the distance of the centre of mass above the horizontal surface. [4]

[You may use without proof the result for the centre of mass of a circular sector in the list of formulae (MF19).]

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The lamina is on the point of toppling about B.

(b) Find the value of k. [2]

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4 A particle Q is initially positioned at a distance d vertically above a particle P. Particle P is projected with speed U at an angle α above the horizontal. At the same time, Q is projected at an angle β below the horizontal. Both particles move freely under gravity. The particles collide at time T after the projections.

(a) Show that $d = UT(\sin\alpha + \cos\alpha\tan\beta)$. [4]

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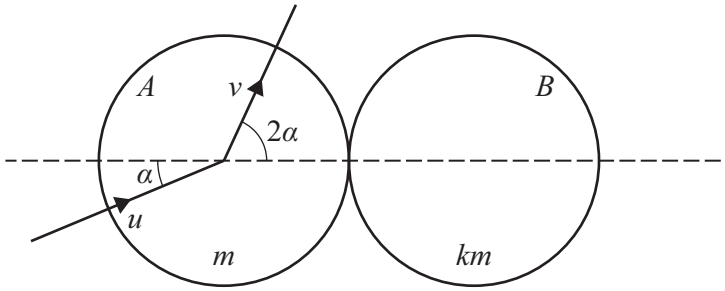
The particles collide when P is at its maximum height.

(b) Given that $\alpha = 30°$ and $\beta = 60°$, find d in terms of U and g. [3]

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5 Two uniform smooth spheres, A and B, of equal radii are on a horizontal surface. They have masses of m and km respectively. Sphere A is moving with speed u at an angle α with the line of centres when it collides with sphere B which is stationary. Immediately after the collision, sphere A moves with speed v at an angle 2α with the line of centres (see diagram).



It is given that $\tan\alpha = \frac{3}{4}$.

(a) Find v in terms of u. [2]

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(b) Find the coefficient of restitution between the spheres in terms of k. [4]

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(c) Find the range of possible values of k. [3]

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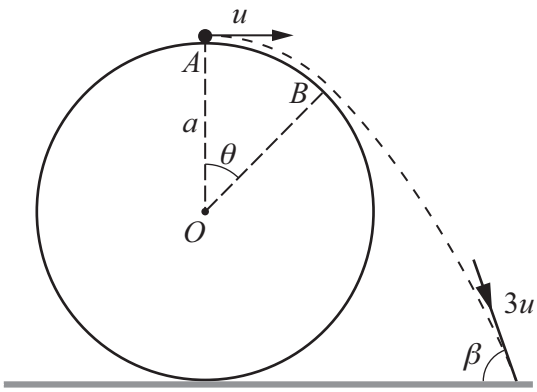
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A fixed smooth sphere with radius a and centre O rests on horizontal ground. A particle is projected horizontally from the highest point, A, of the sphere with speed u. The particle begins to move in a vertical circle along the surface of the sphere. The particle loses contact with the sphere at the point B, where the angle AOB is θ.

After leaving the surface of the sphere, the particle moves freely under gravity before striking the horizontal ground with speed $3u$ at an angle β to the horizontal (see diagram).

Find the value of β. [8]

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7 A particle P of mass $m$kg moving along a rough horizontal table has displacement xm from a fixed point O on the table and velocity $v\,\mathrm{m\,s^{-1}}$ at time ts. The particle P is subject to a resistive force of magnitude $mgkv$N, where k is a positive constant, and a frictional force of magnitude μmg. The particle P is initially at O with speed $U\,\mathrm{m\,s^{-1}}$.

(a) Show that $t = \frac{1}{gk}\ln\left(\frac{kU+\mu}{kv+\mu}\right)$. [4]

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It is given that $U = 10$, $k = 0.04$ and $\mu = 0.2$.

(b) Find the distance P moves before coming to rest. [4]

(c) Find the average speed of P over the period it is moving. [2]

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Additional page

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