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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0295790866

FURTHER MATHEMATICS **9231/41**

Paper 4 Further Probability & Statistics **October/November 2025**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

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1 A group of 10 school children are asked to estimate the size of an angle $\theta°$ in a given acute angled triangle. These estimates, in degrees, are as follows.

84 85 77 85 84 87 86 88 83 85

(a) Stating any assumptions you make, calculate a 95% confidence interval for θ. [5]

(b) Give a reason why the assumptions made in part **(a)** may not be appropriate in this case. [1]

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2 The manager of a car park claims that the number of cars entering the car park follows a Poisson distribution with mean 2.8. The numbers of cars entering the car park are recorded on a working day during successive 5-minute periods. The following table contains the observed frequencies, together with most of the expected frequencies and their contributions to the χ^2-test statistic.

Number of cars	0	1	2	3	4	5	$\geqslant 6$
Observed frequency	2	15	31	29	13	3	7
Expected frequency	6.081	17.03	23.84	p	15.57	8.721	6.511
χ^2-test statistic	2.739	0.241	2.152	q	0.425	3.753	0.037

(a) Find the value of p and the value of q. [2]

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(b) Carry out a goodness of fit test at the 5% significance level to investigate the manager's claim. [4]

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3 A random sample of 10 newborn baby boys is taken and their masses in kg are recorded. From this sample, the population standard deviation of all newborn baby boys is estimated as 0.6 kg. A random sample of 5 newborn baby girls is taken and their masses in kg are recorded as follows.

3.9 3.1 2.9 3.1 3.6

It is assumed that the masses of newborn baby boys and girls have the same population standard deviation, σ kg.

By pooling the two samples, calculate an estimate of σ. [4]

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4 A researcher believes that the median m of a population has changed from its known previous value m_0. The researcher collects a random sample of size 28. She ranks the data and calculates a test statistic T using the Wilcoxon signed-rank test. The conclusion of the test carried out at a 1% significance level is that there is not sufficient evidence to support her belief.

Using a normal approximation, find the least possible value of T. [5]

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5 A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{16}\sqrt{x} & 0 \leqslant x < 4, \\ \frac{1}{k\sqrt{x}} & 4 \leqslant x \leqslant 9, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = 3$. [2]

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(b) Find the median value of X. [3]

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The random variable Y is defined by $Y = \sqrt{X}$.

(c) Find the probability density function of Y. [5]

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6 Nine athletes in a club have a new coach. The coach adopts a new training programme which he believes will reduce the race times of these athletes. Each athlete completes a 1500 m time trial before and after completing the new training programme. Their times, in seconds (s), are recorded.

Athlete	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
Time before training (s)	250	251	252	267	276	291	310	320	335
Time after training (s)	245	251	253	261	275	293	302	313	320

(a) Carry out a paired *t*-test at the 5% significance level to test the coach's belief. [7]

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Further research suggests that the effects of the training programme tend to reduce the times of the slower athletes by more than those of the faster athletes.

(b) Suggest a reason why the paired *t*-test used in part **(a)** may not have been an appropriate test in this case. [1]

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(c) Suggest a suitable alternative test that could have been used instead of a paired *t*-test. [1]

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7 A discrete random variable X takes values $r = 0, 1, 2$ with probabilities $\mathrm{P}(X = r)$ as given in the following table.

r	0	1	2
$\mathrm{P}(X = r)$	a	$2a$	b

(a) Write down the probability generating function of X, and use it to find an expression for $\mathrm{E}(X)$ in terms of a and b. [2]

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(b) Show that $\mathrm{Var}(X) = 2b + 2(a + b)(1 - 2a - 2b)$. [3]

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The random variable Y is defined by $Y = X_1 + X_2 + X_3 + \cdots + X_{10}$ where $X_1, X_2, X_3, \ldots, X_{10}$ are ten independent observations of X.

(c) Using the probability generating function of Y, and your answer to part **(a)**, show that $\mathrm{E}(Y) = 10\mathrm{E}(X)$. [3]

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(d) For the case $b = 0$, define fully the distribution of Y. [2]

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