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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8577139748

FURTHER MATHEMATICS **9231/44**

Paper 4 Further Probability & Statistics **October/November 2025**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

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1 Local residents are concerned about the speed of cars travelling through their village. They record the speed, in km h^{-1}, of a random sample of 13 cars travelling through their village. The recorded speeds are as follows.

40 53 59 42 43 48 62 67 46 82 66 45 70

(a) Construct a 90% confidence interval for the population mean speed of cars passing through the village. [5]

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(b) State an assumption that is necessary for the confidence interval found in **1(a)** to be valid. [1]

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2 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{4}{9}(x+1) & 0 \leqslant x \leqslant 1, \\ (x-2)^2 & 1 < x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

(b) Find the exact value of the median of X. [2]

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The random variable Y is defined by $Y = \sqrt{X}$.

(c) Find the cumulative distribution function of Y. [3]

(d) Determine whether the median of Y is greater than, or less than, the median of X. [2]

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3 Two different types of juice extractor, machine *X* and machine *Y*, are being compared. The manufacturer claims that machine *Y* extracts more juice per orange on average than machine *X*.

A random sample of 20 oranges is selected and each carefully measured. Oranges of a similar size and mass are then paired and numbered from 1 to 10. One orange from each pair is randomly allocated to machine *X* with the other allocated to machine *Y*. The amount of juice, in ml, extracted from each orange is recorded in the following table.

	pair									
	1	2	3	4	5	6	7	8	9	10
machine *X*	65	73	58	61	72	79	64	65	69	71
machine *Y*	68	72	64	63	75	82	63	63	72	74

(a) Use a *t*-test to test the manufacturer's claim at the 1% significance level. [7]

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(b) State an assumption required for the test in **3(a)** to be valid. [1]

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The manufacturer notices that the amount of juice extracted from the oranges in pair 4 were recorded incorrectly. In fact, the amount of juice extracted from machine *X* was 60 ml and from machine *Y* was 62 ml.

(c) Explain, with justification, whether the conclusion of the test in **3(a)** remains the same. [2]

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4 The random variable X takes values 1 and 2 with probabilities $\frac{2}{5}$ and $\frac{3}{5}$ respectively.

(a) Write down the probability generating function $G_X(t)$ of X. [1]

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The random variable Y is the sum of four independent observations of X.

(b) Find the probability generating function $G_Y(t)$ of Y. Give your answer in the form $G_Y(t) = at^m(b+ct)^n$, where a, b, c, m and n are constants to be determined. [2]

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(c) Use $G_Y(t)$ to find $P(Y = 6)$. [2]

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(d) Find Var(Y). [5]

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5 A driving instructor believes that the performance (pass or fail) of students when taking a driving test is associated with their age. The following table summarises the number of students who pass and who fail, and the ages in years of the students taking the test, over a period of three years.

	age of student			
	under 20	20–40	over 40	total
pass	34	41	6	81
fail	16	39	9	64
total	50	80	15	145

Test, at the 10% significance level, whether performance is independent of age of student. [7]

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6 Students of the same age from two schools, school *A* and school *B*, take a large number of quizzes throughout the year and are each awarded a mark out of 1000. The marks of 123 students in school *A* and 147 students in school *B* are ranked from lowest (rank 1) to highest (rank 270). The sum of the ranks of the students from school *A* is 15 355.

(a) Carry out a Wilcoxon rank-sum test at the 5% significance level to investigate whether there is a difference in average marks between the students in school *A* and school *B*. [6]

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(b) State an assumption that is required for the Wilcoxon rank-sum test to be valid. [1]

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