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CAMBRIDGE
International Education

# Cambridge International AS & A Level

**FURTHER MATHEMATICS** **9231/11**

Paper 1 Further Pure Mathematics 1 **May/June 2026**

MARK SCHEME

Maximum Mark: 75

**Published**

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

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**General marking principles**

All examiners must apply these marking principles when marking candidate responses.

| | |
|---|---|
| 1 | Examiners must award marks for each response in line with:<br><br>• the specific **content** defined in the mark scheme<br>• the specific **skills** defined in the mark scheme or in the level descriptions<br>• the **standard of response** required as exemplified by the standardisation scripts. |
| 2 | Examiners must award **whole marks** (not half marks, or other fractions). |
| 3 | Examiners must award marks **positively**. This means that:<br><br>• marks are not deducted for errors or omissions<br>• marks are awarded for correct/valid answers as defined in the mark scheme and also for **other valid answers** which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate<br>• the quality of spelling, punctuation and grammar are judged **only** when the mark scheme indicates that these features are **specifically assessed** in the question. However, the meaning of all responses must be unambiguous. |
| 4 | Examiners must **consistently** apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly. |
| 5 | Examiners must use the **full range of marks** defined in the mark scheme, unless limited by the quality of the candidate responses seen. |
| 6 | Examiners must award marks according to the mark scheme and **must not** award marks with grade thresholds or grade descriptions in mind. |

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**Mathematics-Specific Marking Principles**

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

**Annotations**

| Annotation | Meaning |
|---|---|
|  | More information required |
|  | Accuracy mark awarded zero |
|  | Accuracy mark awarded one |
|  | Independent accuracy mark awarded zero |
|  | Independent accuracy mark awarded one |
|  | Independent accuracy mark awarded two |
|  | Benefit of the doubt |
|  | Blank Page |
|  | Incorrect |
| Dep | Used to indicate DM0 or DM1 |

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| Annotation | Meaning |
|---|---|
| DM1 | Dependent on the previous M1 mark(s) |
| FT | Follow through |
| ~~~ | Indicate working that is right or wrong |
| Highlighter | Highlight a key point in the working |
| ISW | Ignore subsequent work |
| J | Judgement |
| JU | Judgement |
| M0 | Method mark awarded zero |
| M1 | Method mark awarded one |
| M2 | Method mark awarded two |
| MR | Misread |
| O | Omission or Other solution |
| Off-page comment | Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to. |
| On-page comment | Allows comments to be entered in speech bubbles on the candidate response. |
| PE | Judgment made by the PE |
| Pre | Premature approximation |
| SC | Special case |
| SEEN | Indicates that work/page has been seen |

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| Annotation | Meaning |
|---|---|
| SF | Error in number of significant figures |
| ✓ | Correct |
| TE | Transcription error |
| XP | Correct answer from incorrect working |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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9231/11 Mark Scheme June 2026

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $(4r-3)(12r+3)=48r^2-24r-9$ | **B1** | |
| | $48\left(\frac{1}{6}n(n+1)(2n+1)\right)-24\left(\frac{1}{2}n(n+1)\right)-9n$ | **M1** | Substitutes formulae for $\sum r^2$ and $\sum r$. |
| | $=16n^3+12n^2-13n$ | **A1** | CAO Must be in this form. |
| | | **3** | |
| 1(b) | $\frac{1}{(4r-3)(12r+3)}=\frac{1}{12}\left(\frac{1}{4r-3}-\frac{1}{4r+1}\right)$ | **B1** | Finds partial fractions. Accept equivalent form. |
| | $\sum_{r=1}^{n}\frac{1}{(4r-3)(12r+3)}=\frac{1}{12}\left(\frac{1}{1}-\frac{1}{5}\right)+\frac{1}{12}\left(\frac{1}{5}-\frac{1}{9}\right)+\frac{1}{12}\left(\frac{1}{9}-\frac{1}{13}\right)$ | **M1** | Write enough correct <u>sections</u> of terms so that all that are left in the answer appear plus one pair that cancel. |
| | $+\ldots+\frac{1}{12}\left(\frac{1}{4n-3}-\frac{1}{4n+1}\right)$ | **A1** | All necessary <u>complete</u> terms to demonstrate the cancellation. Either two at the beginning and one at the end or one at the beginning and two at the end. |
| | $=\frac{1}{12}-\frac{1}{12(4n+1)}$ | **B1** | Accept any equivalent form. |
| | | **4** | |
| 1(c) | $\sum_{r=1}^{\infty}\frac{1}{(4r-3)(12r+3)}=\frac{1}{12}$ | **B1** | FT their answer for 1(b) of similar form. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | **EITHER**<br>$3^6 + 92 + 89 = 910$ is divisible by 91. | **(B1** | Checks base case. |
| | Assume that $3^{6k} + 92^k + 89$ is divisible by 91 for some positive integer $k$. | **B1** | States inductive hypothesis, making it clear it is an assumption.<br>B0 if assume true for ALL values of $k$. |
| | Then $3^{6(k+1)} + 92^{k+1} + 89 = (728+1)3^{6k} + (91+1)92^k + 89$ | **M1** | |
| | $= 728 \times 3^{6k} + 91 \times 92^k + (3^{6k} + 92^k + 89)$ | **A1** | Separates $3^{6k} + 92^k + 89$ |
| | is divisible by 91 because both $728 \times 3^{6k} + 91 \times 92^k$ and $3^{6k} + 92^k + 89$ are divisible by 91. | **A1** | Connects the divisibility with the hypothesis. |
| | Hence, [by induction,] $3^{6n} + 92^n + 89$ is divisible by 91 for every positive integer $n$. | **A1)** | Must have previous three marks and state the result in full. |
| | **OR**<br>**Alternative method for 2** | | |
| | $3^6 + 92 + 89 = 910$ is divisible by 91. | **(B1** | Checks base case. |
| | Assume that $(3^{6k} + 92^k + 89)$ is divisible by 91 for some positive integer $k$. | **B1** | States inductive hypothesis, making it clear it is an assumption.<br>B0 if assume true for ALL values of $k$. |
| | Then $3^{6(k+1)} + 92^{k+1} + 89 - (3^{6k} + 92^k + 89)$ | **M1** | Considers difference between consecutive terms. |
| | $= (728)3^{6k} + (91)92^k$<br>$= 91(8 \times 3^{6k} + 92^k)$ | **A1** | |
| | | **A1** | Connects the divisibility with the hypothesis. |

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9231/11 Mark Scheme June 2026

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Hence, [by induction,] $3^{6n}+92^{n}+89$ is divisible by 91 for every positive integer $n$. | **A1)** | Must have previous three marks and state the result in full. |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ | **B1** | Stretch parallel to the $x$-axis, scale factor $k$. |
| | $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ | **B1** | Shear, $x$-axis fixed, with $(0,1)$ mapped to $(2,1)$. |
| | $\mathbf{M}=\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ | **M1** | Correct order. |
| | $\mathbf{M}=\begin{pmatrix} 1\times k+2\times 0 & 1\times 0+2\times 1 \\ 0\times k+1\times 0 & 0\times 0+1\times 1 \end{pmatrix}=\begin{pmatrix} k & 2 \\ 0 & 1 \end{pmatrix}$. | **A1** | AG Some intermediate step must be shown. At least one element not in final form. |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(b) | $\begin{pmatrix} k & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx+2y \\ y \end{pmatrix}$ | **B1** | Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ Must see matrix applied to both coordinates. |
| | $kx+2y=x$ | **M1** | Uses $\mathrm{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$. Must be considering a fixed point. |
| | $y=\frac{1}{2}(1-k)x$ | **A1** | |
| | | **3** | |
| 3(c) | Area of P = $\lvert\det M\rvert$ × Area of S $\Rightarrow 1=\frac{1}{2}\lvert k\rvert$ | **M1** | Uses $\det M = k$ with P and S the correct way round. |
| | $k=\pm 2$ | **A1** | CAO |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | **EITHER**<br>$y = x^{-2} \Rightarrow x = y^{-\frac{1}{2}}$<br>$\Rightarrow 5y^{-\frac{3}{2}} - 2y^{-\frac{1}{2}} + 1 = 0 \left[\Rightarrow y^{\frac{3}{2}} = 2y - 5\right]$ | **(B1** | Correct substitution seen used in equation. |
| | $y^3 = (2y-5)^2$ | **M1** | Squares to eliminate fractional powers. |
| | $y^3 - 4y^2 + 20y - 25 = 0$ | **A1)** | Must be an equation with like terms collected. |
| | **OR**<br>**Alternative for 4(a)** | | |
| | $(5x^2 - 2)x = -1$<br>$(5x^2 - 2)^2 x^2 = 1$ | **(M1** | Squares to eliminate odd powers of $x$. |
| | $(5y^{-1} - 2)^2 y^{-1} = 1$ | **B1** | Correct substitution used in the equation. |
| | $y^3 - 4y^2 + 20y - 25 = 0$ | **A1)** | Must be an equation with positive powers and like terms collected. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | $\alpha^{-2}+\beta^{-2}+\gamma^{-2}=4\ \ \alpha^{-2}\beta^{-2}+\beta^{-2}\gamma^{-2}+\gamma^{-2}\alpha^{-2}=20$ | **B1 FT** | Using their answer to part (a). Must be using a cubic equation with positive powers and like terms collected. |
| | $\alpha^{-4}+\beta^{-4}+\gamma^{-4}=4^2-2(20)$ | **M1** | $\alpha^{-4}+\beta^{-4}+\gamma^{-4}=(\alpha^{-2}+\beta^{-2}+\gamma^{-2})^2-2(\alpha^{-2}\beta^{-2}+\beta^{-2}\gamma^{-2}+\gamma^{-2}\alpha^{-2})$ |
| | $=-24$ | **A1 FT** | FT from a cubic equation answer for 4(a) |
| | | **3** | |
| 4(c) | $S_{-6}=4S_{-4}-20S_{-2}+75$ | **M1** | Using $\alpha^{-6}-4\alpha^{-4}+20\alpha^{-2}-25=0$ Must use $S_0=3$<br>FT for their cubic from 4(a) only if using this method. |
| | $S_{-6}=-101$ | **A1** | CAO |
| | $S_{-8}=4S_{-6}-20S_{-4}+25S_{-2}$ | **M1** | Using $\alpha^{-8}-4\alpha^{-6}+20\alpha^{-4}-25\alpha^{-2}=0$<br>FT for their cubic from 4(a) only if using this method. |
| | $S_{-8}=176$ | **A1** | CAO |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | **EITHER**<br>$\begin{pmatrix}-3\\-2\\4\end{pmatrix}-\begin{pmatrix}-2\\4\\4\end{pmatrix}=\begin{pmatrix}-1\\-6\\0\end{pmatrix}$ | **(B1** | Finds direction of one line to another. |
| | $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\3 & -4 & 2\\2 & 4 & 3\end{vmatrix}=\begin{pmatrix}-20\\-5\\20\end{pmatrix}\sim\begin{pmatrix}-4\\-1\\4\end{pmatrix}$ | **M1** | Find common perpendicular by taking cross product of the two correct direction vectors. |
| | | **A1** | Correct. |
| | $\frac{1}{\sqrt{33}}\left\|\begin{pmatrix}-1\\-6\\0\end{pmatrix}\bullet\begin{pmatrix}-4\\-1\\4\end{pmatrix}\right\|=\frac{10}{\sqrt{33}}$ | **M1** | Uses formula for shortest distance using *their* common perpendicular and direction vector. Scalar product and magnitudes must be evaluated. |
| | | **A1)** | Correct answer. Accept 1.74 |
| | **OR**<br>**Alternative method for 5(a)** | | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| | $\begin{pmatrix}-3+2\mu\\-2+4\mu\\4+3\mu\end{pmatrix}-\begin{pmatrix}-2+3\lambda\\4-4\lambda\\4+2\lambda\end{pmatrix}=\begin{pmatrix}-1+2\mu-3\lambda\\-6+4\mu+4\lambda\\0+3\mu-2\lambda\end{pmatrix}$<br><br>$\begin{pmatrix}-1+2\mu-3\lambda\\-6+4\mu+4\lambda\\0+3\mu-2\lambda\end{pmatrix}\bullet\begin{pmatrix}2\\4\\3\end{pmatrix}=0$<br><br>$-29\lambda+4\mu=-21$<br>$4\lambda+29\mu=26$ | **(M1** | Vector joining P on one line to Q on the other line, perpendicular to one line direction. |
| 5(a) | $\lambda=\frac{101}{165}\quad\mu=\frac{139}{165}$ | **A1** | Solves equations from using perpendicular to both lines to find $\lambda$ and $\mu$ |
| | $\begin{pmatrix}-\frac{40}{33}\\-\frac{10}{33}\\\frac{40}{33}\end{pmatrix}$ | **M1** | Finds PQ |
| | $PQ=\sqrt{\left(-\frac{40}{33}\right)^2+\left(-\frac{10}{33}\right)^2+\left(\frac{40}{33}\right)^2}$ | **M1** | Finds length of PQ |
| | $=\frac{10}{\sqrt{33}}$ | **A1)** | |
| | | **5** | |

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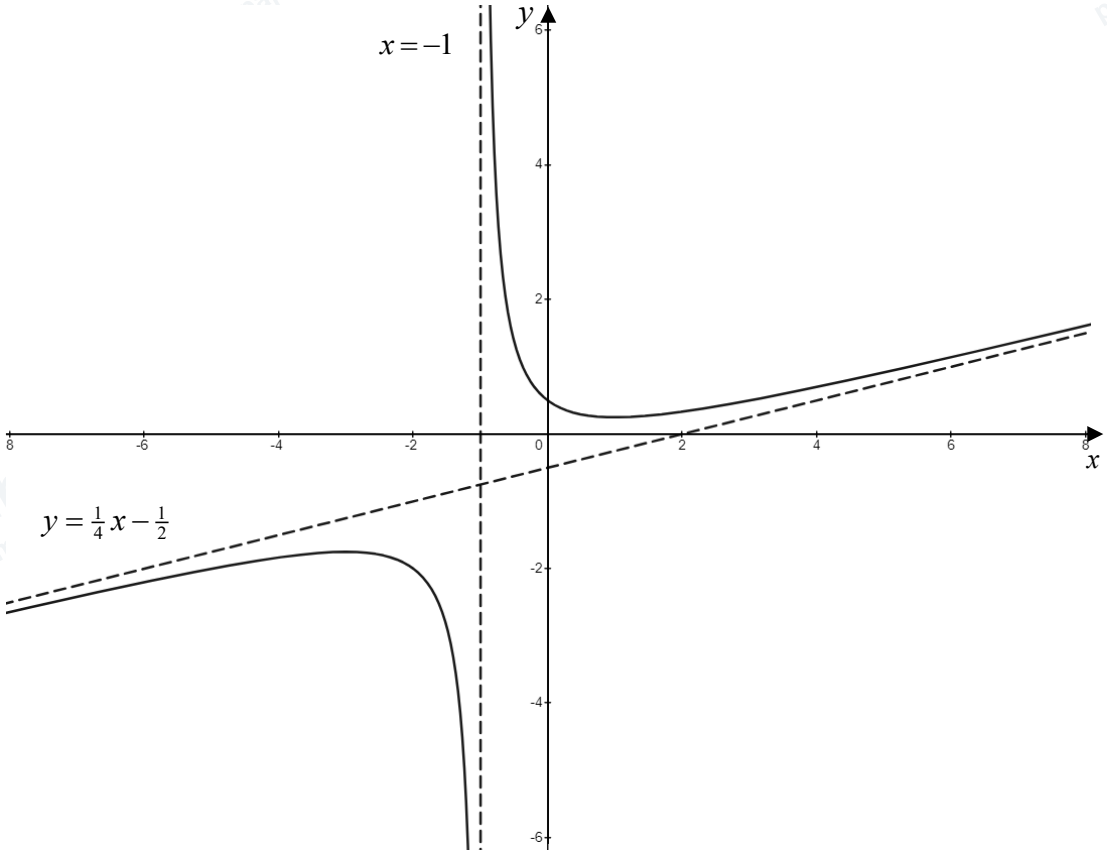
| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 5 & 3 \\ 3 & -4 & 2 \end{vmatrix} = \begin{pmatrix} 22 \\ 15 \\ -3 \end{pmatrix}$ | *M1 | Finds the normal to $\Pi_1$ by taking cross product of two correct direction vectors. |
| | | A1 | Correct. |
| | $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = \begin{pmatrix} -22 \\ 17 \\ -8 \end{pmatrix}$ | *M1 | Finds the normal to $\Pi_2$ by taking cross product of two correct direction vectors. |
| | | A1 | Correct |
| | $\begin{pmatrix} 22 \\ 15 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} -22 \\ 17 \\ -8 \end{pmatrix} = \sqrt{718}\sqrt{837}\cos\theta \Rightarrow \cos\theta = \dfrac{-205}{\sqrt{718}\sqrt{837}}$ | DM1 | Uses dot product of their normal vectors. Scalar product and magnitudes must be evaluated. |
| | $180 - \theta = 74.7^\circ$ | A1 | Or 1.30 rad. Mark final answer. AWRT 74.7 (1dp) or 1.3 (1dp) |
| | | 6 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $x = -1$ | B1 | Vertical asymptote. |
| | $y = \dfrac{(x+1)(x-2)+4}{4x+4}$ | M1 | Oblique asymptote - must show attempt to find constant term. |
| | $y = \frac{1}{4}x - \frac{1}{2}$ | A1 | Must be a linear equation. |
| | | 3 | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | $\frac{dy}{dx} = \frac{4x^2 + 8x - 12}{(4x+4)^2} = 0$<br>$4x^2 + 8x - 12 = 0$ | **M1** | Sets $\frac{dy}{dx} = 0$. |
| | $\left(1, \frac{1}{4}\right),\ \left(-3, -\frac{7}{4}\right)$ | **A1** | A1 for $x$-coordinates. |
| | | **A1** | A1 for $y$-coordinates correctly linked to $x$-coordinates.<br>Allow A1 for one completely correct point. |
| | | **3** | |

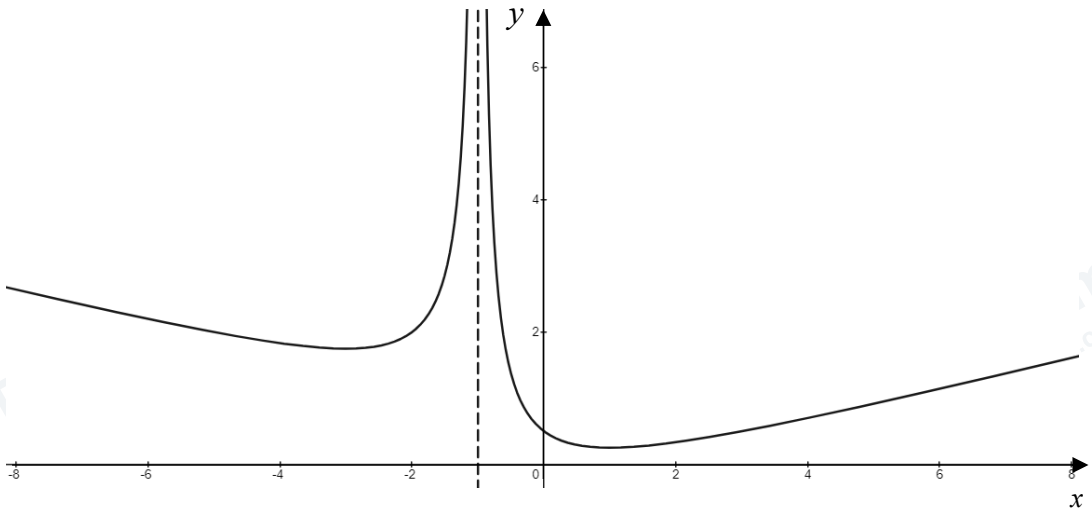
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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(c) |  | **B1 FT** | Axes and asymptotes labelled.<br>Follow through their two asymptotes from (a) for this mark only. |
| | | **B1** | Left branch correct and also all approaches to the asymptotes. |
| | | **B1** | Right branch correct including a smooth minimum in the first quadrant. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(d) |  | **B1** | Must be fully correct, including vertical asymptote. |
| | | **1** | |
| 6(e) | $\frac{x^2-x+2}{4x+4}=-2$ or $\frac{x^2-x+2}{4x+4}=2$<br>$x^2+7x+10=0$ $x^2-9x-6=0$ | **M2** | Finds critical points, award M1 for each case. Must be using equations. |
| | $x=-5,-2$ $x=\frac{1}{2}\left(9-\sqrt{105}\right), x=\frac{1}{2}\left(9+\sqrt{105}\right)$ | **A1** | All four correct, exact answers. |
| | $-5<x<-2$ $\frac{1}{2}\left(9-\sqrt{105}\right)<x<\frac{1}{2}\left(9+\sqrt{105}\right)$ | **A1** | CAO |
| | | **4** | |

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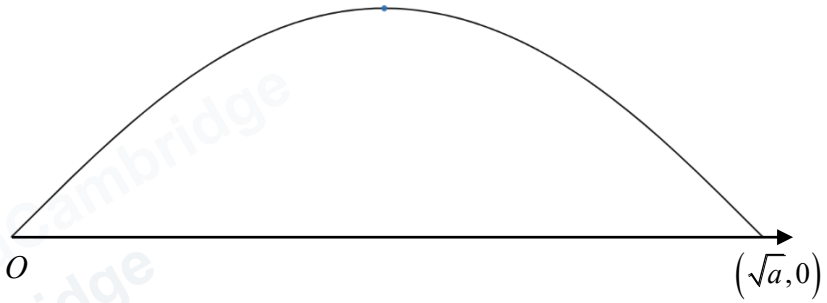
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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | **EITHER**<br>$y=\left(\frac{a}{1+\tan 2\theta}\right)^{\frac{1}{2}}\sin\theta$ | **(B1** | Uses $y=r\sin\theta$ |
| | $\left[\frac{\mathrm{d}y}{\mathrm{d}\theta}\right]=\left(\frac{a}{1+\tan 2\theta}\right)^{\frac{1}{2}}\cos\theta-\frac{1}{2}\sin\theta\left(\frac{a}{1+\tan 2\theta}\right)^{-\frac{1}{2}}\frac{\left(2a\sec^2 2\theta\right)}{\left(1+\tan 2\theta\right)^2}$ | ***M1** | Differentiates using product rule |
| | $\left[\frac{\mathrm{d}y}{\mathrm{d}\theta}\right]=\frac{\left(a\left(1+\tan 2\theta\right)\right)^{\frac{1}{2}}\cos\theta-\frac{1}{2}\left(\frac{a}{1+\tan 2\theta}\right)^{-\frac{3}{2}}\left(2a\sec^2 2\theta\right)\sin\theta}{1+\tan 2\theta}$ | **A1** | or quotient rule. Must be using correct function. |
| | $\Rightarrow\cos\theta-\frac{\sec^2 2\theta\sin\theta}{1+\tan 2\theta}=\cos\theta-\frac{\sin\theta}{\cos 2\theta\left(\sin 2\theta+\cos 2\theta\right)}=0$ | **DM1 A1** | Puts derivative = 0 and rearranges.<br><br>AG. Intermediate steps must be shown. |
| | $\cos(0.5)-\frac{\sin(0.5)}{\cos(2\times0.5)\left(\sin(2\times0.5)+\cos(2\times0.5)\right)}=0.235$ and<br>$\cos(0.6)-\frac{\sin(0.6)}{\cos(2\times0.6)\left(\sin(2\times0.6)+\cos(2\times0.6)\right)}=-0.379$ | **B1)** | Shows sign change. Values calculated to at least one decimal place. |
| | **OR**<br>**Alternative method for 7(a)** | | |
| | $y^2=\left(\frac{a}{1+\tan 2\theta}\right)\sin^2\theta$ | **(B1** | Uses $y=r\sin\theta$ |
| | $\left[2y\frac{\mathrm{d}y}{\mathrm{d}\theta}\right]=\left(\frac{1}{1+\tan 2\theta}\right)2a\sin\theta\cos\theta-a\sin^2\theta\left(\frac{1}{1+\tan 2\theta}\right)^{-2}\sec^2 2\theta$ | ***M1** | Differentiates using product rule |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $\left[2y\frac{\mathrm{d}y}{\mathrm{d}\theta}\right]=\left((1+\tan 2\theta)2a\sin\theta\cos\theta-2a\sin^2\theta\sec^2 2\theta\right)\left(\frac{1}{1+\tan 2\theta}\right)^2$ | **A1** | or quotient rule. Must be using correct function. |
| | $\Rightarrow\cos\theta-\frac{\sec^2 2\theta\sin\theta}{1+\tan 2\theta}=\cos\theta-\frac{\sin\theta}{\cos 2\theta(\sin 2\theta+\cos 2\theta)}=0$ | **DM1** | Puts derivative = 0 and rearranges. |
| | | **A1** | AG. Intermediate steps must be shown. |
| | $\cos(0.5)-\frac{\sin(0.5)}{\cos(2\times0.5)(\sin(2\times0.5)+\cos(2\times0.5))}=0.235$ and<br>$\cos(0.6)-\frac{\sin(0.6)}{\cos(2\times0.6)(\sin(2\times0.6)+\cos(2\times0.6))}=-0.379$ | **B1)** | Shows sign change. Values calculated to at least one decimal place. |
| | | **6** | |
| 7(b) |  | **B1** | Initial line and pole clear. Curve in first quadrant through $(\sqrt{a},0)$ and $O$.<br>Condone pole not labelled but must be clearly identifiable. $(\sqrt{a},0)$ not needed. |
| | | **B1** | Correct shape (gradient at end points and approximately symmetrical) Maximum point should lie between 40% and 60% of width of the arc. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(c) | $\frac{1}{2}\int_0^{\frac{1}{4}\pi} r^2\,d\theta = \frac{a}{2}\int_0^{\frac{1}{4}\pi} \frac{1}{1+\tan 2\theta}\,d\theta$ | **B1** | Using correct formula and correct limits. |
| | $v = \tan 2\theta \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}\theta} = 2\sec^2 2\theta$ | **M1** | Applies given substitution. Condone $\frac{\mathrm{d}v}{\mathrm{d}\theta} = \sec^2 2\theta$ |
| | $\frac{a}{2}\int_0^{\frac{1}{4}\pi} \frac{1}{1+\tan 2\theta}\,d\theta = \frac{a}{4}\int_0^{\infty} \frac{1}{1+v}\frac{1}{\sec^2 2\theta}\,dv$ | **M1** | Clearly applies $\sec^2 2\theta = 1+\tan^2 2\theta$<br>Allow $\sec^2 2\theta = 1+v^2$ written explicitly. |
| | $= \frac{a}{4}\int_0^{\infty} \frac{1}{1+v}\frac{1}{1+\tan^2 2\theta}\,dv = \frac{a}{4}\int_0^{\infty} \frac{1}{(1+v)(1+v^2)}\,dv$ | **A1** | Some working must be shown, including calculations supporting change of limits.<br>AG. |
| | $\frac{1}{(1+v)(1+v^2)} = \frac{1}{2}\left(\frac{1}{v+1} - \frac{v-1}{v^2+1}\right)$ | ***M1** | Partial fractions of correct form. |
| | $= \frac{1}{2}\left(\frac{1}{v+1} - \frac{v}{v^2+1} + \frac{1}{v^2+1}\right)$ | **A1** | and separates $\frac{-v+1}{v^2+1}$. |
| | $\frac{a}{8}\left[\ln(v+1) - \frac{1}{2}\ln(v^2+1) + \tan^{-1} v\right]_0^{\infty}$ | **DM1** | Applies all three formulae from MF19 to a three-term expression from above.<br>Follow through their constants from line above. |
| | | **A1** | Correct. |
| | $\frac{a}{8}\left[\ln\left(\frac{v+1}{\sqrt{v^2+1}}\right) + \tan^{-1} v\right]_0^{\infty} = \frac{1}{16}\pi a$ | **A1** | The four previous marks must have been awarded and logarithms must be combined for this mark to be awarded. |
| | | **9** | |

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