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International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 9 9 0 5 2 1 7 3 4 0 *

FURTHER MATHEMATICS **9231/13**

Paper 1 Further Pure Mathematics 1 **May/June 2026**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 The cubic equation $x^3 + bx^2 + cx + d = 0$, where b, c and d are constants, has roots α, β and γ . It is given that

$$\begin{aligned} \alpha \quad + \beta \quad + \gamma \quad &= 5, \\ \alpha^2 \quad + \beta^2 \quad + \gamma^2 \quad &= -19, \\ \alpha^{-1} + \beta^{-1} + \gamma^{-1} &= 11. \end{aligned}$$

(a) Find the values of b, c and d. [5]

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(b) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. [2]

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2 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix **M** represents a sequence of two geometrical transformations in the x-y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

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(b) Find the equations of the invariant lines, through the origin, of the transformation represented by **M**. [5]

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3 The matrices **A**, **P** and **D** are given by

$$\mathbf{A} = \begin{pmatrix} 8 & -5 \\ 7 & -4 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 5 & 3 \\ 7 & 3 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}.$$

Prove by mathematical induction that, for all positive integers n, $\mathbf{A}^n = \mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$. [7]

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4 **(a)** By considering $(2r+1)^4-(2r-1)^4$, use the method of differences to prove that

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2.$$ [6]

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Let $S_n = 1^3 - 2^3 + 3^3 - 4^3 + \ldots + (-1)^{n+1} n^3$.

(b) Show that $S_{2n} = n^2(an + b)$, where a and b are integers to be determined. [3]

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5 The curve C has polar equation $r^2 = \theta \sec^2 \theta$, where $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole. [2]

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(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{3}\pi$. [5]

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(c) Show that C has Cartesian equation $y = x\tan(x^2)$, where $0 \leqslant x \leqslant \frac{1}{3}\sqrt{3\pi}$. [3]

(d) Using your answer to **5(b)**, deduce the exact value of $\int_0^{\frac{1}{3}\sqrt{3\pi}} x\tan(x^2)\,\mathrm{d}x$. [3]

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6 The curve C has equation $y = \dfrac{x+1}{x^2+x+1}$.

(a) Show that C has no vertical asymptotes and state the equation of the horizontal asymptote. [2]

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(b) Find the coordinates of any stationary points on C. [3]

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(c) Sketch C, stating the coordinates of the intersections with the axes. [3]

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(d) Sketch the curve with equation $y = \left|\frac{x+1}{x^2+x+1}\right|$. [2]

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(e) Find in exact form the set of values of x for which $\left|\frac{x+1}{x^2+x+1}\right| < \frac{1}{3}$. [4]

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7 The plane Π has equation $\mathbf{r} = \mathbf{j} + 2\mathbf{k} + s(\mathbf{i} - \mathbf{j}) + t(-3\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(a) Obtain an equation of Π in the form $ax + by + cz = d$. [4]

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(b) The point P has position vector $(6 + 6\theta)\mathbf{i} + (1 + \theta)\mathbf{j} + (2 + 5\theta)\mathbf{k}$.

Find the set of values of θ for which the perpendicular distance of P from Π is less than $\sqrt{6}$. [4]

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(c) Find an equation for the common perpendicular to the lines $\mathbf{r} = \lambda(\mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 2\mathbf{k} + \mu(-3\mathbf{i} + \mathbf{j} - \mathbf{k})$. [8]

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