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CANDIDATE NAME

CENTRE NUMBER

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FURTHER MATHEMATICS **9231/14**

Paper 1 Further Pure Mathematics 1 **May/June 2026**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 **(a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n}(r-1)^2 = an^3 + bn^2 + cn,$$

where a, b and c are constants to be determined. [2]

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It is given that $u_r = \frac{1}{r^2-2r+1} - \frac{1}{r^2+2r+1}$.

(b) Use the method of differences to find $\sum_{r=2}^{n} u_r$ in terms of n. [4]

(c) State the value of $\sum_{r=2}^{\infty} u_r$. [1]

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2 The equation $2x^4 + 3x^3 + x + 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find a quartic equation whose roots are $\frac{1}{\alpha^3}, \frac{1}{\beta^3}, \frac{1}{\gamma^3}, \frac{1}{\delta^3}$ and state the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3} + \frac{1}{\delta^3}$. [5]

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(b) Find the value of $\frac{1}{\alpha^6} + \frac{1}{\beta^6} + \frac{1}{\gamma^6} + \frac{1}{\delta^6}$. [2]

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3 The sequence of positive numbers $u_1, u_2, u_3, \ldots$ is such that $u_1 > 1$ and, for $n \geqslant 1$,

$$u_{n+1} = \frac{u_n^2 + 2u_n + 2}{u_n + 4}.$$

(a) By considering $u_{n+1} - 1$, or otherwise, prove by mathematical induction that $u_n > 1$ for all positive integers n. [5]

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(b) Show that $u_{n+1} < u_n$ for $n \geqslant 1$. [3]

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4 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 1 & k & -1 \\ 3 & 1 & k \\ 3 & 1 & -1 \end{pmatrix},$$

where k is a real constant.

(a) Find the values of k for which **A** is singular. [3]

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The matrices **B** and **C** are given by

$$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

(b) Show that $\mathbf{CAB} = \begin{pmatrix} k+3 & -1 \\ 2 & -k-1 \end{pmatrix}$. [3]

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(c) It is given that $k = -2$

Show that the transformation in the x-y plane represented by **CAB** has no invariant lines. [5]

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5 The points A, B, C have position vectors

$$4\mathbf{i}+3\mathbf{j}+3\mathbf{k}, \quad 2\mathbf{i}+2\mathbf{j}+\mathbf{k}, \quad 5\mathbf{i}+2\mathbf{j}+2\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

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The point D has position vector $2\mathbf{i}+\mathbf{j}+3\mathbf{k}$.

(b) Find the acute angle between the line OD and the plane ABC. [3]

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(c) Find the shortest distance between the lines AB and CD. [5]

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6 The curve C has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{2x^2 - x + 1}{2x^2 - 2x}$.

(a) Find the equations of the asymptotes of C. [2]

(b) Find the coordinates of any stationary points on C, giving your answers correct to 1 decimal place. [4]

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(c) Sketch C. [3]

(d) Sketch the curve with equation $y = \frac{1}{\mathrm{f}(x)}$ and state the set of values for which $\frac{1}{\mathrm{f}(x)} < \mathrm{f}(x)$. [4]

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7 **(a)** Show that the curve with Cartesian equation

$$\left(x^2+y^2\right)^{\frac{3}{2}} = xy$$

has polar equation $r = \frac{1}{2}\sin 2\theta$. [3]

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The curves C_1 and C_2 have polar equations

$$r = \tfrac{1}{2}\sin 2\theta \quad \text{and} \quad r^2 = \frac{1}{\left(\theta - \frac{1}{4}\pi\right)^2 + 5}$$

respectively, where $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(b) Show that C_1 and C_2 intersect where

$$\left(\left(\theta - \tfrac{1}{4}\pi\right)^2 + 5\right)\sin^2 2\theta = 4$$

and verify by calculation that this equation has a root α between $\theta = 0.5$ and $\theta = 0.6$. [3]

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(c) Sketch C_1 and C_2 on the same diagram, clearly identifying each curve and any lines of symmetry. [4]

(d) The region R is enclosed by C_1 and C_2 and includes the pole. Show that the area of R is given by

$$\tfrac{1}{32}\left(4\alpha - \sin 4\alpha\right) + \tfrac{1}{5}\sqrt{5}\tan^{-1}\left(\tfrac{1}{5}\sqrt{5}\left(\tfrac{1}{4}\pi - \alpha\right)\right),$$

where α is the root found in **7(b)**. [6]

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Additional page

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