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CAMBRIDGE
International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **May/June 2026**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

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General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: • the specific **content** defined in the mark scheme • the specific **skills** defined in the mark scheme or in the level descriptions • the **standard of response** required as exemplified by the standardisation scripts.
2	Examiners must award **whole marks** (not half marks, or other fractions).
3	Examiners must award marks **positively**. This means that: • marks are not deducted for errors or omissions • marks are awarded for correct/valid answers as defined in the mark scheme and also for **other valid answers** which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate • the quality of spelling, punctuation and grammar are judged **only** when the mark scheme indicates that these features are **specifically assessed** in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must **consistently** apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the **full range of marks** defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and **must not** award marks with grade thresholds or grade descriptions in mind.

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Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
FT	Follow through
~~	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
J	Judgement
JU	Judgement
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
MR	Misread
O	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
PE	Judgment made by the PE
Pre	Premature approximation
SC	Special case
SEEN	Indicates that work/page has been seen

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Annotation	Meaning
SF	Error in number of significant figures
✓	Correct
TE	Transcription error
XP	Correct answer from incorrect working

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$f'(x)=\left(1+(x+1)^2\right)^{-\frac{1}{2}}$	**B1**	Finds first derivative. Accept $\frac{1}{x+1+\sqrt{(x+1)^2+1}}\left(1+\frac{2x+2}{2\sqrt{(x+1)^2+1}}\right)$ or implicit $y'\cosh y=1$.
	$f''(x)=-\left(1+(x+1)^2\right)^{-\frac{3}{2}}(x+1)$	**B1**	Finds second derivative. Allow not simplified or implicit $y''\cosh y+(y')^2\sinh y=0.$
	$\sinh^{-1}1=\ln\left(1+\sqrt{2}\right)$	**B1**	
	$f'(0)=\frac{1}{\sqrt{2}}\quad f''(0)=-\frac{1}{2\sqrt{2}}$	**M1**	Evaluates their derivatives at zero only. Must have attempted both derivatives.
	$\sinh^{-1}(x+1)=\ln\left(1+\sqrt{2}\right)+\frac{1}{\sqrt{2}}x-\frac{1}{4\sqrt{2}}x^2$	**A1**	CWO.
		5	

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Question	Answer	Marks	Guidance
2(a)	$I_n = \int_0^1 (1+t)^n \sin \pi t \, \mathrm{dt} = \left[-\frac{1}{\pi}(1+t)^n \cos \pi t \right]_0^1 + \frac{n}{\pi}\int_0^1 (1+t)^{n-1} \cos \pi t \, \mathrm{d}t$	*M1 A1	Applies integration by parts. Allow limits missing. M1 for correct trigonometric functions and powers of $(1+t)$.
	$I_n = \frac{1}{\pi}\left(2^n+1\right) + \frac{n}{\pi}\left[\frac{1}{\pi}(1+t)^{n-1} \sin \pi t \right]_0^1 - \frac{n}{\pi}\int_0^1 \frac{1}{\pi}(n-1)\left(1+t\right)^{n-2} \sin \pi t \, \mathrm{d}t$	DM1	Applies integration by parts again. M1 for correct trigonometric functions and powers of $(1+t)$.
	$I_n = \frac{1}{\pi}\left(2^n+1\right) - \frac{1}{\pi^2}n(n-1)I_{n-2} \Rightarrow \pi^2 I_n = \pi\left(2^n+1\right) - n(n-1)I_{n-2}$	A1	AG. Must have seen limits and correct use of brackets. CWO.
		4	
2(b)	$I_0 = \int_0^1 \sin \pi t \ \mathrm{d}t = \left[-\frac{1}{\pi}\cos \pi t \right]_0^1 = \frac{2}{\pi}$.	B1	Finds I_0.
	$\pi^2 I_2 = \pi\left(2^2+1\right) - 2(2-1)I_0 \ \left[= 5\pi - \frac{4}{\pi} \right]$	M1	Uses reduction formula from **(a)**.
	$I_2 = \frac{5}{\pi} - \frac{4}{\pi^3}$	A1	
		3	

Question	Answer	Marks	Guidance
3(a)	$u = \cosh x \Rightarrow \mathrm{d}u = \sinh x \, \mathrm{d}x$	B1	
	$\int \sinh^2 x \, \mathrm{d}u = \int \left(u^2 - 1\right) \mathrm{d}u$	M1 A1	Applies correct identity $\sinh^2 u = \cosh^2 u - 1$ to find integral in terms of u.
	$= \frac{1}{3}u^3 - u \quad (+C) \ = \frac{1}{3}\cosh^3 x - \cosh x \quad (+C)$	A1	
		4	

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Question	Answer	Marks	Guidance
3(b)	$e^{\int \coth x \, dx} = e^{\ln \sinh x} = \sinh x$	**M1 A1**	Finds integrating factor.
	$\frac{d}{dx}(y \sinh x) = \sinh^3 x$	**M1**	Correct form on LHS and attempt to integrate RHS.
	$y \sinh x = \frac{1}{3}\cosh^3 x - \cosh x + C$	***M1 A1**	Integrates RHS using their part **(a)**. For M1, their answer to **(a)** must be of the correct form $a\cosh^3 x + b\cosh x$ with a, b non-zero.
	$0 = \frac{1}{3}\left(\frac{5}{3}\right)^3 - \frac{5}{3} + C \Rightarrow C = \frac{10}{81}$	**DM1**	Substitutes initial conditions. M0 if expression incorrectly manipulated before substitution. E.g. not all terms divided by $\sinh x$.
	$y = \text{cosech} x\left(\frac{1}{3}\cosh^3 x - \cosh x + \frac{10}{81}\right)$	**A1**	OE.
		7	

Question	Answer	Marks	Guidance
4(a)	$\frac{d}{dx}\left((x-y)^5\right) = 5(x-y)^4(1-y')$	**B1**	Differentiates $(x-y)^5$ correctly.
	$\frac{d}{dx}(\ln(xy)) = \frac{xy'+y}{xy} = y'y^{-1} + x^{-1}$	**B1**	Differentiates $\ln(xy)$ correctly.
	$5(1-y') - \frac{1}{2}y' - 1 = 0 \Rightarrow y' = \frac{8}{11}$	**B1**	Substitutes $(-1,-2)$ into $5(x-y)^4(1-y') + y'y^{-1} + x^{-1} = 0$, AG. CWO. Might see $y'\left(y^{-1} - 5(x-y)^4\right) = -5(x-y)^4 - x^{-1}$.
		3	

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Question	Answer	Marks	Guidance
4(b)	**EITHER:** $-5(x-y)^4 y"+20(1-y')^2(x-y)^3$	**(B1 B1**	Differentiates $5(x-y)^4(1-y')$.
	$-(y')^2 y^{-2}+y^{-1}y"-x^{-2}=0$	**B1**	Differentiates $y'y^{-1}+x^{-1}$. Might see $\frac{xy'+y}{xy}$ differentiated using product or quotient rules. E.g. $\frac{xy(xy"+2y')-(xy'+y)^2}{(xy)^2}$. Or $-(xy)^{-2}(xy'+y)^2+(xy)^{-1}(xy"+2y')$.
	$-5(1)^4 y"+20(1-\frac{8}{11})^2(1)^3-\frac{1}{4}(\frac{8}{11})^2-\frac{1}{2}y"-1=0$	**M1**	Substitutes $(-1,-2)$ and $y'=\frac{8}{11}$.
	$-\frac{11}{2}y"+\frac{43}{121}=0\Rightarrow y"=\frac{86}{1331}$	**A1)**	
	OR: **Alternative method for question 4(b)**		
	$y'(-y^{-2}y'-20(x-y)^3(1-y'))+y"(y^{-1}-5(x-y)^4)$	**(B1 B1**	Differentiates both sides of $y'(y^{-1}-5(x-y)^4)=-5(x-y)^4-x^{-1}$.
	$=-20(x-y)^3(1-y')+x^{-2}$	**B1**	
	$(\frac{8}{11})(-\frac{1}{4}\times\frac{8}{11}-20\times\frac{3}{11})+y"(-\frac{1}{2}-5)=-20\times\frac{3}{11}+1$	**M1**	Substitutes $(-1,-2)$ and $y'=\frac{8}{11}$.
	$-\frac{11}{2}y"+\frac{43}{121}=0\Rightarrow y"=\frac{86}{1331}$	**A1)**	

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Question	Answer	Marks	Guidance
4(b)	**OR:** **Second alternative method for question 4(b)**		
	$\left(5(x-y)^4-y^{-1}\right)\left(20(x-y)^3(1-y')-x^{-2}\right)$ $-\left(5(x-y)^4+x^{-1}\right)\left(20(x-y)^3(1-y')+y^{-2}y'\right)$	**(B1 B1**	Numerator of derivative of $y'=\dfrac{5(x-y)^4+x^{-1}}{5(x-y)^4-y^{-1}}$.
	$\left(5(x-y)^4-y^{-1}\right)^2$	**B1**	Denominator of derivative of $y'=\dfrac{5(x-y)^4+x^{-1}}{5(x-y)^4-y^{-1}}$. Can be implied by $\left(\frac{11}{2}\right)^2$.
	$y''=\dfrac{\left(5+\frac{1}{2}\right)\left(20\times\frac{3}{11}-1\right)-(5-1)\left(20\times\frac{3}{11}+\frac{2}{11}\right)}{\left(5+\frac{1}{2}\right)^2}$	**M1**	Substitutes $(-1,-2)$ and $y'=\frac{8}{11}$ into their expression.
	$y''=\frac{86}{1331}$	**A1)**	
	Available marks	**5**	

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Question	Answer	Marks	Guidance
5(a)	$\int_0^1 \left(\frac{11}{10}x^3+\frac{1}{10}\right)\mathrm{d}x \leqslant \left(\frac{1}{n}\right)\left(\frac{11}{10}\left(\frac{1}{n}\right)^3+\frac{1}{10}\right)+\left(\frac{1}{n}\right)\left(\frac{11}{10}\left(\frac{2}{n}\right)^3+\frac{1}{10}\right)+\cdots+\left(\frac{1}{n}\right)\left(\frac{11}{10}\left(\frac{n}{n}\right)^3+\frac{1}{10}\right)$	**M1 A1**	Forms the sum of the areas of the rectangles. M1 for sum of n complete terms. A1 for at least 3 correct terms including first and last. May be written in summation form.
	$=\frac{11}{10n^4}\sum_{r=1}^{n} r^3+\frac{1}{10}=\frac{11n^2(n+1)^2}{40n^4}+\frac{1}{10}$	**M1**	Applies formula for $\sum r^3$ from MF19.
	$=\frac{11}{40}\left(1+\frac{1}{n}\right)^2+\frac{1}{10}$	**A1**	AG. CWO.
		4	
5(b)	$\int_0^1 \left(\frac{11}{10}x^3+\frac{1}{10}\right)\mathrm{d}x \geqslant \frac{1}{10n}+\left(\frac{1}{n}\right)\left(\frac{11}{10}\left(\frac{1}{n}\right)^3+\frac{1}{10}\right)+\left(\frac{1}{n}\right)\left(\frac{11}{10}\left(\frac{2}{n}\right)^3+\frac{1}{10}\right)+\cdots+\left(\frac{1}{n}\right)\left(\frac{11}{10}\left(\frac{n-1}{n}\right)^3+\frac{1}{10}\right)$	**M1 A1**	Forms the sum of the areas of appropriate rectangles. M1 for sum of n complete terms. A1 for at least 3 correct terms including first and last.
	$=\frac{1}{10n}+\frac{11}{10n^4}\sum_{r=1}^{n-1} r^3+\frac{n-1}{10n}=\frac{11(n-1)^2n^2}{40n^4}+\frac{1}{10}$	**M1**	Applies formulae from MF19. Allow $U_n+\frac{1}{10n}-\frac{12}{10n}$
	$=\frac{11}{40}\left(1-\frac{1}{n}\right)^2+\frac{1}{10}$	**A1**	OE. e.g. $\frac{11}{40}\left(1+\frac{1}{n}\right)^2+\frac{1}{10}-\frac{11}{10n}$.
		4	
5(c)	$U_n-L_n=\frac{11}{40}\left(1+\frac{1}{n}\right)^2-\frac{11}{40}\left(1-\frac{1}{n}\right)^2$	**M1**	Their U_n-L_n simplifies to $\frac{c}{n}$. Accept $\frac{11}{40}(1)^2+\frac{1}{10}-\frac{11}{40}(1)^2-\frac{1}{10}$.
	$\frac{11}{10n}\to 0$ as $n\to\infty$	**A1**	AG. Their L_n must be correct.
		2	

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Question	Answer	Marks	Guidance
6	$5m^2+4m+1=0 \Rightarrow m=-\frac{2}{5}\pm\frac{1}{5}\mathrm{i}$	**M1**	Auxiliary equation. M1 for correct type of roots $a\pm b\mathrm{i}$.
	$[y=]\mathrm{e}^{-\frac{2}{5}x}\left(A\cos\frac{1}{5}x+B\sin\frac{1}{5}x\right)$	**A1**	Complementary function. Allow '$y=$' missing.
	$y=px^2+qx+r \Rightarrow y'=2px+q \Rightarrow y''=2p$	**B1**	Particular integral and its derivatives.
	$p=1\ \ 8p+q=7\ \ 10p+4q+r=1$	**M1**	Substitutes and equates coefficients.
	$q=-1\ \ r=-5$	**A1**	
	$y=\mathrm{e}^{-\frac{2}{5}x}\left(A\cos\frac{1}{5}x+B\sin\frac{1}{5}x\right)+x^2-x-5$	**A1**	General solution. Must have '$y=$'. FT on CF.
	$y'=\mathrm{e}^{-\frac{2}{5}x}\left(-\frac{1}{5}A\sin\frac{1}{5}x+\frac{1}{5}B\cos\frac{1}{5}x\right)-\frac{2}{5}\mathrm{e}^{-\frac{2}{5}x}\left(A\cos\frac{1}{5}x+B\sin\frac{1}{5}x\right)+2x-1$	**M1**	Differentiates. General solution must be of the correct form. Must use product rule.
	$0=A-5\ \ 0=\frac{1}{5}B-\frac{2}{5}A-1\ \Rightarrow A=5, B=15$	**DM1 A1**	Uses initial conditions. A1 is both A and B correct.
	$y=\mathrm{e}^{-\frac{2}{5}x}\left(5\cos\frac{1}{5}x+15\sin\frac{1}{5}x\right)+x^2-x-5$	**A1**	Must have '$y=$'. CWO.
		10	

Question	Answer	Marks	Guidance
7(a)	$\frac{z-z^{n+1}}{1-z}$ or $\frac{z^{n+1}-z}{z-1}$ or $\frac{z\left(z^n-1\right)}{z-1}$	**B1**	
		1	

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Question	Answer	Marks	Guidance
7(b)	$e^{i\frac{2}{7}\pi}$	**B1**	States one root of $z^7=1,\ z\neq 1$.
	$e^{i\frac{2k}{7}\pi}\quad k=2,\ldots,6$	**B1**	Five other roots. No repeats or incorrect roots, $z=1$ must be omitted.
		2	
7(c)	**EITHER:** $\sum_{m=1}^{\infty} z^m = \frac{z}{1-z} = \frac{e^{-1+i\theta}}{1-e^{-1+i\theta}}$	**(*M1 A1**	Applies sum to infinity and substitutes for z. Expect $\frac{e^{i\theta}}{e-e^{i\theta}}$.
	$= \frac{e^{-1+i\theta}\left(1-e^{-1-i\theta}\right)}{\left(1-e^{-1+i\theta}\right)\left(1-e^{-1-i\theta}\right)} = \frac{e^{-1+i\theta}-e^{-2}}{1-e^{-1-i\theta}-e^{-1+i\theta}+e^{-2}}$	**DM1**	Rationalises denominator. Expect $\frac{e^{i\theta}}{e-e^{i\theta}}\left(\frac{e-e^{-i\theta}}{e-e^{-i\theta}}\right) = \frac{e^{1+i\theta}-1}{e^2-e\left(e^{i\theta}+e^{-i\theta}\right)+1}$.
	$= \frac{e^{-1}\left(\cos\theta+i\sin\theta\right)-e^{-2}}{1-2e^{-1}\cos\theta+e^{-2}}\quad\left[=\frac{e\left(\cos\theta+i\sin\theta\right)-1}{e^2-2e\cos\theta+1}\right]$	**DM1 A1**	Applies $e^{i\theta}+e^{-i\theta}=2\cos\theta$
	$\mathrm{Re}\left(\sum_{m=1}^{\infty} z^m\right) = \frac{e^{-1}\cos\theta-e^{-2}}{1-2e^{-1}\cos\theta+e^{-2}} = \frac{e\cos\theta-1}{e^2-2e\cos\theta+1}$	**DM1 A1)**	Takes the real part, AG.
	OR: **Alternative method for question 7(c)**		
	$\sum_{m=1}^{\infty} z^m = \frac{z}{1-z} = \frac{e^{i\theta}}{e-e^{i\theta}} = \frac{\cos\theta+i\sin\theta}{\left(e-\cos\theta\right)-i\sin\theta}$	**(*M1 A1**	Applies sum to infinity and substitutes for z.

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Question	Answer	Marks	Guidance
7(c)	$\dfrac{\cos\theta+\mathrm{i}\sin\theta}{(\mathrm{e}-\cos\theta)-\mathrm{i}\sin\theta}\left(\dfrac{(\mathrm{e}-\cos\theta)+\mathrm{i}\sin\theta}{(\mathrm{e}-\cos\theta)+\mathrm{i}\sin\theta}\right)$ $=\dfrac{\mathrm{e}\cos\theta-\cos^2\theta+\mathrm{i}\sin\theta\cos\theta+\mathrm{i}\sin\theta(\mathrm{e}-\cos\theta)-\sin^2\theta}{(\mathrm{e}-\cos\theta)^2+\sin^2\theta}$	**DM1**	Rationalises denominator.
	$=\dfrac{\mathrm{e}\cos\theta-1+\mathrm{ie}\sin\theta}{\mathrm{e}^2-2\mathrm{e}\cos\theta+1}$	**DM1 A1**	Applies $\sin^2\theta+\cos^2\theta=1$.
	$\mathrm{Re}\left(\sum_{m=1}^{\infty}z^m\right)=\dfrac{\mathrm{e}\cos\theta-1}{\mathrm{e}^2-2\mathrm{e}\cos\theta+1}$	**DM1 A1)**	Takes the real part, AG.
	Available marks	**7**	

Question	Answer	Marks	Guidance
8(a)	$\begin{vmatrix}-1 & -2 & 1\\ a & 1 & -1\\ 0 & b & 2\end{vmatrix}=4a+ab-b-2$	**M1 A1**	Finds determinant in terms of a and b.
	$a=\dfrac{b+2}{b+4}$ [since $b\neq-4$]	**A1**	
		3	

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Question	Answer	Marks	Guidance
8(b)	$(\lambda+1)(\lambda-1)(\lambda-2)=\lambda^3-2\lambda^2-\lambda+2=0$	**B1**	Finds characteristic equation.
	$\mathbf{A}^4-2\mathbf{A}^3-\mathbf{A}^2+2\mathbf{A}=\mathbf{0}$	**M1**	Replaces λ with $\mathbf{A}$ and multiplies through by $\mathbf{A}$.
	$\mathbf{A}^4=2\left(2\mathbf{A}^2+\mathbf{A}-2\mathbf{I}\right)+\mathbf{A}^2-2\mathbf{A}=5\mathbf{A}^2-4\mathbf{I}$	**M1 A1**	Substitutes $\mathbf{A}^3=2\mathbf{A}^2+\mathbf{A}-2\mathbf{I}$. CWO
		4	
8(c)	Eigenvalues of $\mathbf{A}$ are $-1, 1$ and 2.	**B1**	Upper diagonal matrix or characteristic equation.
	$\lambda=-1$: $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\0&-2&1\\0&0&3\end{vmatrix}=\begin{pmatrix}-6\\0\\0\end{pmatrix}\sim\begin{pmatrix}1\\0\\0\end{pmatrix}$	**M1 A1**	Uses vector product (or equations) to find corresponding eigenvectors. M0 if all three vectors are zero vectors.
	$\lambda=1$: $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-2&-2&1\\0&0&-1\end{vmatrix}=\begin{pmatrix}2\\-2\\0\end{pmatrix}\sim\begin{pmatrix}1\\-1\\0\end{pmatrix}$ $\lambda=2$: $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-3&-2&1\\0&-1&-1\end{vmatrix}=\begin{pmatrix}3\\-3\\3\end{pmatrix}\sim\begin{pmatrix}1\\-1\\1\end{pmatrix}$	**A1 A1**	
	Thus $\mathbf{P}=\begin{pmatrix}1&1&1\\0&-1&-1\\0&0&1\end{pmatrix}$ and $\mathbf{D}=\begin{pmatrix}-1&0&0\\0&1&0\\0&0&128\end{pmatrix}$	**M1 A1**	Or correctly matched permutations of columns. A column of zeros or repeated column in $\mathbf{P}$ is M0.
		7	

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